

THE COMPLETELY RANDOMIZED DESIGN

A	B	C	
470	520	610	$k=3$ $n=12$
510	570	550	
540	530	620	
560		590	
580			
$n_1=5$	$n_2=3$	$n_3=4$	
$\bar{x}_1=532$	$\bar{x}_2=540$	$\bar{x}_3=592.5$	
$S_1^2=1870$	$S_2^2=700$	$S_3^2=958.33$	

$$\bar{x} = \frac{\sum_{i=1}^{12} x_i}{12} = \frac{470 + \dots + 590}{12} = 554.17$$

$$\begin{aligned} \boxed{SST} &= \text{Sum of Squares for Treatments} = \\ &= n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + n_3(\bar{x}_3 - \bar{x})^2 \\ &= 5(532 - 554.17)^2 + 3(540 - 554.17)^2 + 4(592.5 - 554.17)^2 \\ &= 8936.67 \end{aligned}$$

$$\begin{aligned} \boxed{SSE} &= \text{Sum of Squares for Error} = \\ &= (n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2 = \\ &= (5 - 1)1870 + (3 - 1)700 + (4 - 1)958.33 \\ &= 11754.99 \end{aligned}$$

$$SS_{\text{total}} = 8936.67 + 11754.99 = 20,691.66$$

MST = Mean Square For Treatments =

$$= \frac{SST}{k-1} = \frac{8936.67}{3-1} = 4468.34$$

MSE = Mean Square For Error =

$$= \frac{SSE}{n-k} = \frac{11754.99}{12-3} = 1306.11$$

$$F = \frac{MST}{MSE} = \frac{4468.34}{1306.11} \approx 3.4$$

ANOVA Summary Table

Sources of Variation	df	SS	MS	F
Treatment	k-1	SST	MST	MST/MSE
Error	n-k	SSE	MSE	/ / /
Total	n-1	SStot	/ / /	/ / / / /

Source	df	SS	MS	F
Treatment	2	8936.67	4468.34	3.4
Error	9	11754.99	1306.11	/ / /
Total	11	20691.66	/ / /	/ / / / /

Step 1 $H_0: \mu_1 = \mu_2 = \mu_3$

H_a : At least two treatment means differ

Step 2 Test statistic

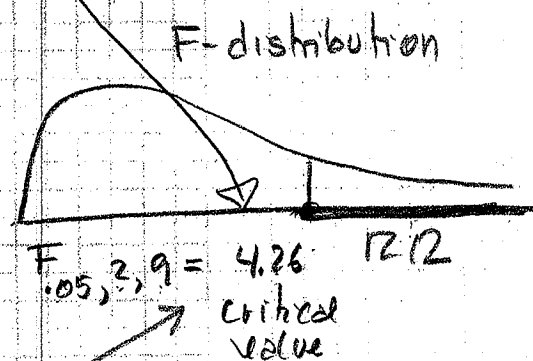
$$F = 3.4$$

Step 3 Rejection Region

$$\alpha = .05$$

$$df_{\text{num}} = v_1 = k - 1 = 2$$

$$df_{\text{denom}} = v_2 = n - k = 9$$



$$RR: F > 4.26$$

Step 4 Decision

Do not reject H_0

Step 5 Conclusion

"the data provide insufficient evidence to conclude that at least two treatment means differ.", at $\alpha = .05$

A	B	C
470	520	610
510	570	550
540	530	620
560		590
580		
<hr/>	<hr/>	<hr/>
$T_1 = 2660$	$T_2 = 1620$	$T_3 = 2370$

$$\sum y = T_1 + T_2 + T_3 = 6650$$

$$CM = \frac{(\sum y)^2}{n} = \frac{6650^2}{12} = 3,685,208.33$$

$$\sum y^2 = 470^2 + 510^2 + \dots + 590^2 = 3,705,900$$

$$SS_{total} = \sum y^2 - CM = 3,705,900 - 3,685,208.33$$

$$= 20,691.67$$

$$SST = \frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \frac{T_3^2}{n_3} - CM =$$

$$= \frac{2660^2}{5} + \frac{1620^2}{3} + \frac{2370^2}{4} - 3,685,208.33$$

$$= 8,936.67$$

$$SS_{total} = SST + SSE$$

$$SSE = SS_{total} - SST$$