

THE COMPLETE RANDOMIZED BLOCK DESIGN

The completely Randomized Design

A	B	C
470	520	610
510	570	550
540	530	620
560		590
580		

$$SST_{tot} = SST + SSE \Rightarrow$$

$$\Rightarrow SSE = SST_{tot} - SST$$

$$SST_{tot} = SST + SSB + SSE \Rightarrow$$

$$\Rightarrow SSE = SST_{tot} - SST - SSB$$

Driver	A	B	C		
1	470	510	520	$B_1 = 1500$	$\bar{X}_{B_1} = 500$
2	530	540	550	$B_2 = 1620$	$\bar{X}_{B_2} = 540$
3	560	570	580	$B_3 = 1710$	$\bar{X}_{B_3} = 570$
4	590	610	620	$B_4 = 1820$	$\bar{X}_{B_4} = 606.7$
	<u>T₁</u>	<u>T₂</u>	<u>T₃</u>		
	2150	2230	2270		$\bar{X} = 554.17$
	\bar{X}_{T_1}	\bar{X}_{T_2}	\bar{X}_{T_3}		
	537.50	557.50	567.50		

(Step 1) $H_0: \mu_{T_1} = \mu_{T_2} = \mu_{T_3}$

H_a : At least two treatment means differ

(Step 2) Test statistic $F = \frac{MST}{MSE}$

$$\begin{aligned}
 SST &= \sum_{l=1}^k b (\bar{x}_{T_l} - \bar{x})^2 = \sum_{l=1}^3 4 (\bar{x}_{T_l} - \bar{x})^2 = \\
 &= 4 (\bar{x}_{T_1} - \bar{x})^2 + 4 (\bar{x}_{T_2} - \bar{x})^2 + 4 (\bar{x}_{T_3} - \bar{x})^2 \\
 &= 4 (537,5 - 554,17)^2 + 4 (557,5 - 554,17)^2 + 4 (567,5 - 554,17)^2 \\
 &= 1,866,67
 \end{aligned}$$

$$\begin{aligned}
 SSB &= \sum_{l=1}^b k (\bar{x}_{B_l} - \bar{x})^2 = \sum_{l=1}^4 3 (\bar{x}_{B_l} - \bar{x})^2 = \\
 &= 3 (\bar{x}_{B_1} - \bar{x})^2 + 3 (\bar{x}_{B_2} - \bar{x})^2 + 3 (\bar{x}_{B_3} - \bar{x})^2 + 3 (\bar{x}_{B_4} - \bar{x})^2 = \\
 &= 3 (500 - 554,17)^2 + 3 (540 - 554,17)^2 + 3 (570 - 554,17)^2 + \\
 &\quad + 3 (600,7 - 554,17)^2 = 18,425
 \end{aligned}$$

$$\begin{aligned}
 SST_{\text{ot}} &= \sum_{l=1}^n (x_l - \bar{x})^2 = \sum_{l=1}^{12} (x_l - \bar{x})^2 \\
 &= (470 - 554,17)^2 + \dots + (620 - 554,17)^2 = 20,691,67
 \end{aligned}$$

$$SST_{\text{ot}} = SST + SSB + SSE \Rightarrow \boxed{SSE = SST_{\text{ot}} - SST - SSB}$$

$$SSE = 20,691,67 - 1,866,67 - 18,425 = 400$$

$$MST = \frac{SST}{k-1} = \frac{1866,67}{3-1} = 933,34$$

$$MSE = \frac{SSE}{n-1 - (k-1) - (b-1)} = \frac{SSE}{n-b-k+1} = \frac{400}{6} = 66,67$$

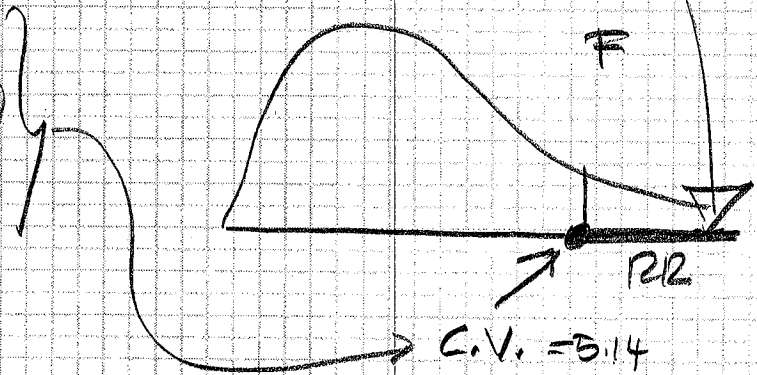
$$F = \frac{MST}{MSE} = \frac{933.34}{66.67} = 14$$

Step 3 Rejection Region

$$\alpha = .05$$

$$df_{num} = v_1 = k - 1 = 2$$

$$df_{denom} = v_2 = 6$$



$$RR: F > 5.14$$

Step 4 Decision

Reject H_0

Step 5 Conclusion

"The data provide sufficient evidence to conclude that at least two gasoline brands differ in the mean miles per gallon, at $\alpha = .05$ "