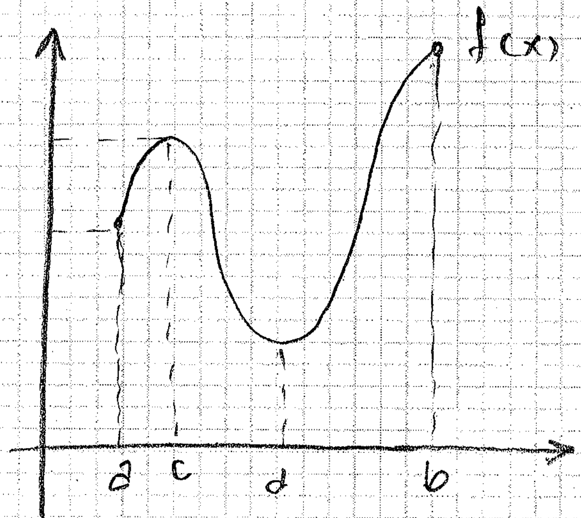
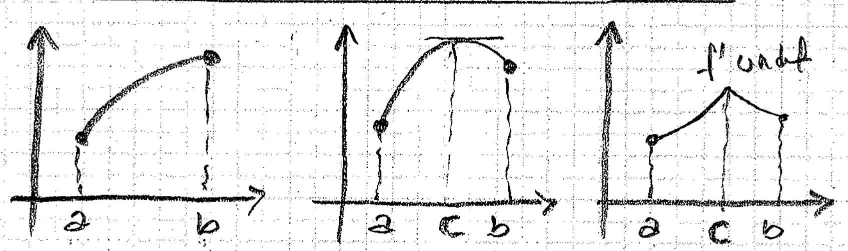
ABSOLUTE MAXIMUM AND MINIMUM



If $f(x_0) \geq f(x)$ for all x in an interval I, we say that f has an absolute maximum at x_0 in I

For Example, if we consider the interval $I = [a, b]$, f has an absolute maximum at $x = b$ and an absolute minimum at $x = d$

ABSOLUTE MAXIMUM AND MINIMUM ON A FINITE CLOSED INTERVAL



If a function f is continuous on a finite closed interval $[a, b]$ then it has both an absolute maximum and an absolute minimum (Extreme Value theorem)

1) Find the critical points in (a, b)
2) Evaluate f at the critical points and the end points
3) the largest value found in step 2 is the absolute maximum and the smallest, the absolute minimum

Example:

$$f(x) = 2x^3 - 21x^2 + 60x$$

on $[1, 6]$

$$f'(x) = 6x^2 - 42x + 60$$
$$= 6(x^2 - 7x + 10)$$
$$= 6(x-2)(x-5)$$

$f'(x) = 0$ $\qquad$ $f'(x)$ undef

$\boxed{x=2}$ $\boxed{x=5}$ $\qquad$ Never

C.P $\qquad$ C.P

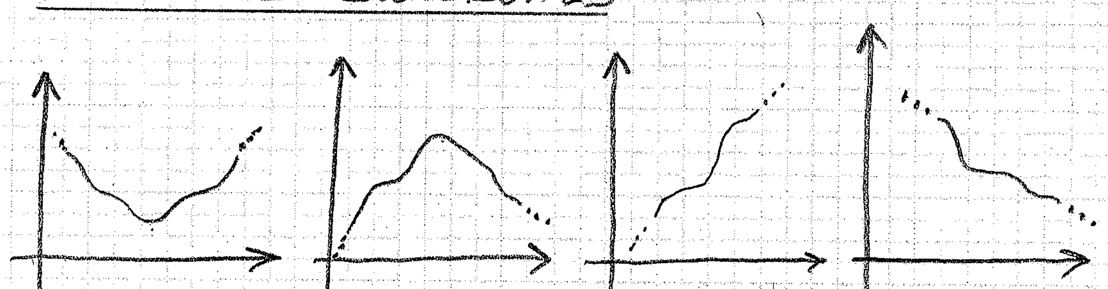
$f(1) = 41$

$f(2) = 52$ ABS MAX

$f(5) = 25$ ABS MIN

$f(6) = 36$

ABSOLUTE MAXIMUM AND MINIMUM ON INFINITE INTERVALS



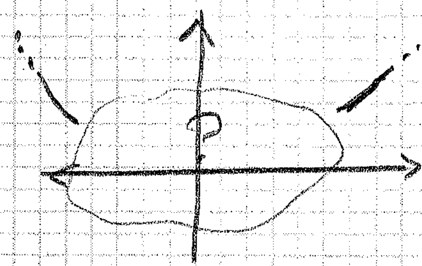
on $(-\infty, \infty)$	on $(-\infty, \infty)$	on $(-\infty, \infty)$	on $(-\infty, \infty)$
$\lim_{x \to -\infty} f(x) = +\infty$	$\lim_{x \to -\infty} f(x) = -\infty$	$\lim_{x \to -\infty} f(x) = -\infty$	$\lim_{x \to -\infty} f(x) = +\infty$
$\lim_{x \to +\infty} f(x) = +\infty$	$\lim_{x \to +\infty} f(x) = -\infty$	$\lim_{x \to +\infty} f(x) = +\infty$	$\lim_{x \to +\infty} f(x) = -\infty$

Example:

$$f(x) = 2x^4 + 3x^3 \quad \text{on } (-\infty, +\infty)$$

$$\lim_{x\to -\infty} (\cancel{2x^4}^{+\infty} + \cancel{3x^3}^{-\infty}) = +\infty$$

$$\lim_{x\to +\infty} (\cancel{2x^4}^{+\infty} + \cancel{3x^3}^{+\infty}) = +\infty$$



$$f'(x) = 8x^3 + 9x^2 = x^2(8x+9)$$

$f'(x) = 0$

$\boxed{x=0}$ C.P $\boxed{x = -9/8}$ C.P

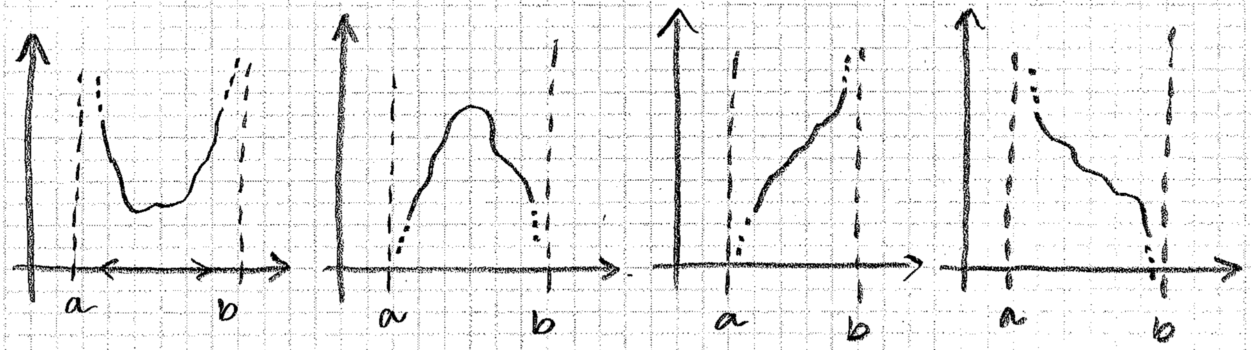
$f'(x)$ undef ?
NONE

$f(0) = 0$

$f(-9/8) = -1.067$ ABS MIN

NO ABS MAX

ABSOLUTE MAXIMUM AND MINIMUM ON OPEN INTERVALS (a,b)



$\lim_{x\to a^+} f(x) = +\infty$ | $\lim_{x\to a^+} f(x) = -\infty$ | $\lim_{x\to a^+} f(x) = -\infty$ | $\lim_{x\to a^+} f(x) = +\infty$

$\lim_{x\to b^-} f(x) = +\infty$ | $\lim_{x\to b^-} f(x) = -\infty$ | $\lim_{x\to b^-} f(x) = +\infty$ | $\lim_{x\to b^-} f(x) = -\infty$

Example: $f(x) = \dfrac{2}{x^2-9x}$ on $(0,9)$

$$\lim_{x\to 0^+} \frac{2}{x^2-9x} = \lim_{x\to 0^+} \frac{2}{\underset{+}{x}\,\underset{-}{(x-9)}} = -\infty$$

$$\lim_{x\to 9^-} \frac{2}{\underset{+}{x}\,\underset{-}{(x-9)}} = -\infty$$

$$f'(x) = \frac{0\cdot(x^2-9x) - 2(2x-9)}{(x^2-9x)^2} = -\frac{2(2x-9)}{(x^2-9x)^2}$$

$f'(x)=0$

$2x-9=0$

$\boxed{x = 9/2}$

C.P

At $x=9/2$ f has the ABS MAX

$f'(x)$ undef

$x^2-9x=0$

$x(x-9)=0$

$\boxed{x=0}$ NO CP

$\boxed{x=9}$ NO CP