

Antiderivatives

The antiderivative of x^2 is the function whose derivative equals x^2

$$\left(\frac{1}{3}x^3\right)' = \frac{1}{3} \cdot 3x^2 = x^2$$

$$\left(\frac{1}{3}x^3 + 2\right)' = \frac{1}{3} \cdot 3x^2 = x^2$$

$$\int x^2 dx = \frac{1}{3}x^3 + C$$

Indefinite Integral

$$\text{EX: } \int e^x dx = e^x + C$$

$$(e^x)' = e^x \quad (e^x + C)' = e^x$$

$$\text{EX: } \int \frac{1}{x} dx = \ln|x| + C$$

$$(\ln x)' = \frac{1}{x}$$

Immediate Integrals

$$1) \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \text{for } n \neq -1$$

$$2) \int x^{-1} dx = \ln|x| + C$$

$$3) \int e^x dx = e^x + C \quad \text{or} \quad \int e^{kx} dx = \frac{e^{kx}}{k} + C$$

$$4) \int b^x dx = \frac{b^x}{\ln b} + C$$

$$5) \int dx = x + C$$

PROPERTIES

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$$\int [f(x) - g(x)] dx = \int f(x) dx - \int g(x) dx$$

$$\int k f(x) dx = k \int f(x) dx$$

EXERCISES:

$$1) \int x^5 dx = \frac{x^6}{6} + C$$

$$2) \int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + C$$

$$= -\frac{1}{2} x^{-2} + C = -\frac{1}{2x^2} + C$$

$$\begin{aligned}
 3) \int \frac{5}{\sqrt[3]{t^2}} dt &= 5 \int t^{-2/3} dt \\
 &= 5 \frac{t^{-2/3+1}}{-2/3+1} + C = \frac{5}{1/3} t^{1/3} + C \\
 &= 15 \sqrt[3]{t} + C
 \end{aligned}$$

$$\begin{aligned}
 4) \int (e^x + 2)^2 dx &= \int (e^{2x} + 2e^x \cdot 2 + 4) dx \\
 \left. \begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ (e^x)^2 &= e^{2x} \end{aligned} \right\} &= \int e^{2x} dx + \int 4e^x dx + \int 4 dx \\
 &= \frac{e^{2x}}{2} + 4e^x + 4x + C
 \end{aligned}$$

$$\begin{aligned}
 5) \int \frac{x^2 + 5x + 6}{x^2} dx &= \int \left(\frac{x^2}{x^2} + \frac{5x}{x^2} + \frac{6}{x^2} \right) dx \\
 &= \int (1 + 5x^{-1} + 6x^{-2}) dx = \int dx + 5 \int x^{-1} dx + 6 \int x^{-2} dx \\
 &= x + 5 \ln|x| + 6 \cdot \frac{x^{-1}}{-1} + C \\
 &= x + 5 \ln|x| - 6x^{-1} + C
 \end{aligned}$$