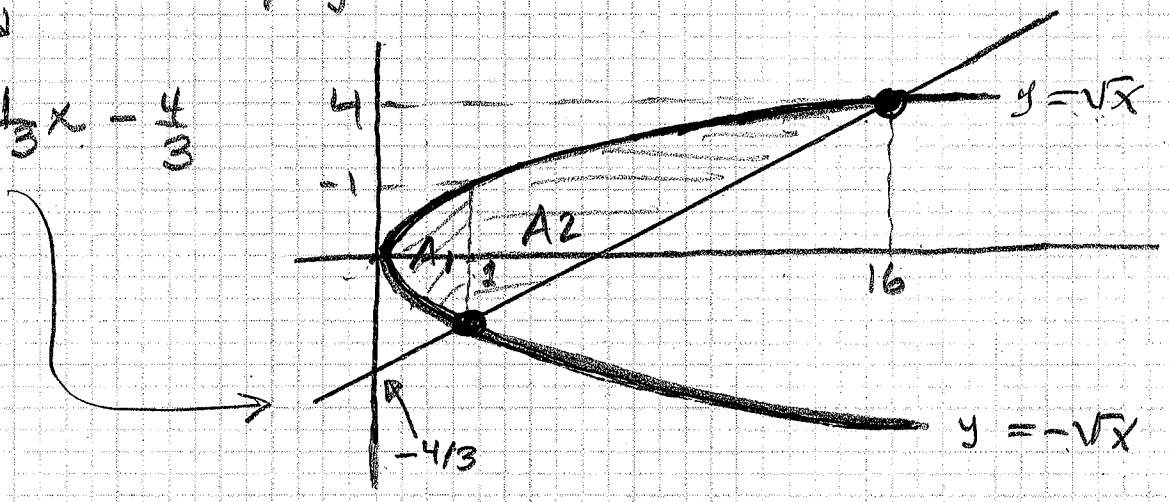
Example Find the area of the region enclosed by

$x = y^2 \Rightarrow \begin{cases} y = \sqrt{x} \\ \text{or} \\ y = -\sqrt{x} \end{cases}$

and

$y = \frac{1}{3}x - \frac{4}{3}$



$A = A_1 + A_2$

$x = y^2$

$y = \frac{1}{3}x - \frac{4}{3} \Rightarrow 3y = x - 4 \Rightarrow x = 3y + 4$ } $\Rightarrow$

$\Rightarrow y^2 = 3y + 4 \Rightarrow y^2 - 3y - 4 = 0 \Rightarrow (y+1)(y-4) = 0$

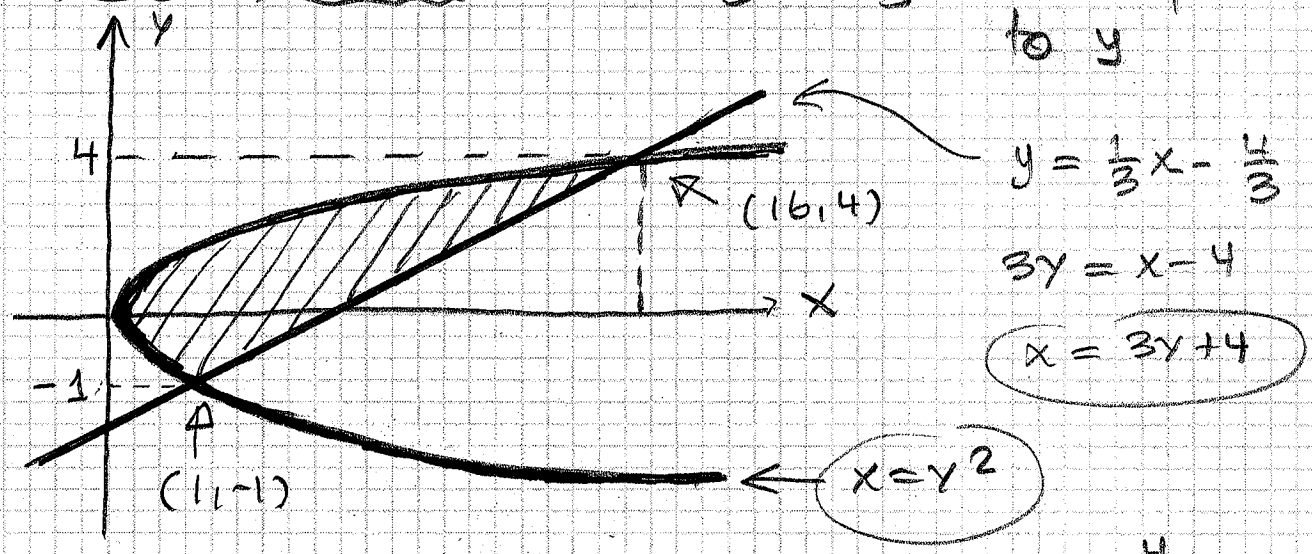
$\Rightarrow y = -1 \Rightarrow x = (-1)^2 = 1 \quad (1, -1)$

$y = 4 \Rightarrow x = 16 \quad (16, 4)$

$$A = \int_0^1 [\sqrt{x} - (-\sqrt{x})]\,dx + \int_1^{16} [\sqrt{x} - (\tfrac{1}{3}x - \tfrac{4}{3})]\,dx$$

$$= \frac{125}{6}$$

SECOND SOLUTION: Integrating with respect to y



$$A = \int_{-1}^{4} (3y + 4 - y^2)\, dy = \left(\frac{3y^2}{2} + 4y - \frac{y^3}{3}\right)\Big|_{-1}^{4}$$

$$= \frac{125}{6}$$