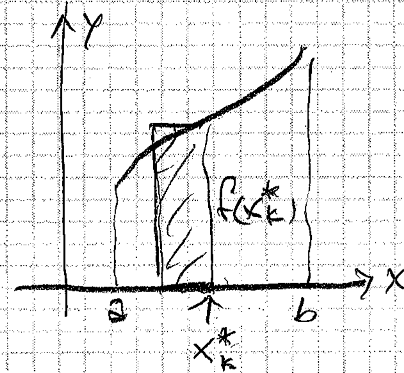
AREA AS A LIMIT



$f(x) \geq 0$

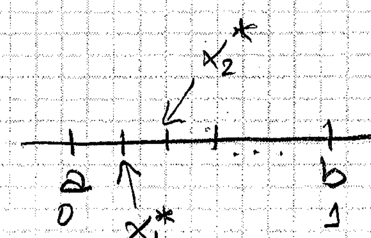
f is continuous in $[a,b]$

$$A = \lim_{n \to +\infty} \sum_{k=1}^{n} f(x_k^*) \cdot \Delta x$$

Ex: $f(x) = x^2$ in $[0,1]$ using right endpoints

Divide $[0,1]$ into n subintervals

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$$



$$x_1^* = a + \Delta x$$
$$= 0 + \frac{1}{n} = \frac{1}{n}$$

$$x_2^* = a + 2\Delta x = 2 \cdot \frac{1}{n} = \frac{2}{n}$$

$\vdots$

$$x_k^* = a + k \cdot \Delta x = k \cdot \frac{1}{n} = \frac{k}{n}$$

$$f(x_k^*) = f\left(\frac{k}{n}\right) = \frac{k^2}{n^2}$$

Area of the k^{th} rectangle

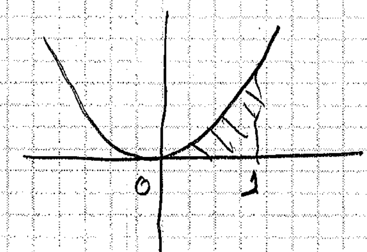
$$\frac{k^2}{n^2} \cdot \frac{1}{n} = \frac{k^2}{n^3}$$

$$A = \lim_{n \to +\infty} \sum_{k=1}^{n} \frac{k^2}{n^3} = \lim_{n \to +\infty} \left[\frac{1}{n^3} \sum_{k=1}^{n} k^2 \right]$$

$$A = \lim_{n \to +\infty} \left[\frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right]$$

$$= \frac{1}{6} \lim_{n \to +\infty} \left[\frac{n(n+1)(2n+1)}{n^3} \right] = \frac{1}{6} \lim_{n \to +\infty} \left[\frac{2n^3 + \cdots}{n^3} \right]$$

$$= \frac{1}{6} \cdot \frac{2}{1} = \frac{1}{3}$$



A = signed area