

Properties of the Sampling Distribution of \bar{x}

| x | $P(x)$ | $x P(x)$ | $x - \mu$ | $(x - \mu)^2$ | $(x - \mu)^2 P(x)$ |
|-----|---------------|---------------|-----------|---------------|----------------------------------|
| 0 | $\frac{1}{4}$ | 0 | -6 | 36 | $\frac{36}{4}$ |
| 6 | $\frac{1}{4}$ | $\frac{6}{4}$ | 0 | 0 | 0 |
| 9 | $\frac{1}{2}$ | $\frac{9}{2}$ | 3 | 9 | $\frac{9}{2}$ |
| | | <u>6</u> | | | <u>$\frac{27}{2}$</u> |

$$\mu = \sum [x P(x)] = 0 * \frac{1}{4} + 6 * \frac{1}{4} + 9 * \frac{1}{2} = 6$$

$$\sigma^2 = \sum [(x - \mu)^2 \cdot P(x)] = \frac{27}{2}$$

| \bar{x} | $P(\bar{x})$ | $\bar{x} P(\bar{x})$ | $\bar{x} - \mu_{\bar{x}}$ | $(\bar{x} - \mu_{\bar{x}})^2$ | $(\bar{x} - \mu_{\bar{x}})^2 P(\bar{x})$ |
|-----------|-----------------|----------------------|---------------------------|-------------------------------|--|
| 0 | $\frac{1}{64}$ | 0 | -6 | 36 | $\frac{36}{64}$ |
| 2 | $\frac{3}{64}$ | $\frac{6}{64}$ | -4 | 16 | $16 * \frac{3}{64}$ |
| 3 | $\frac{6}{64}$ | $\frac{18}{64}$ | -3 | 9 | $9 * \frac{6}{64}$ |
| 4 | $\frac{3}{64}$ | $\frac{12}{64}$ | -2 | 4 | $4 * \frac{3}{64}$ |
| 5 | $\frac{12}{64}$ | $\frac{60}{64}$ | -1 | 1 | $1 * \frac{12}{64}$ |
| 6 | $\frac{13}{64}$ | $\frac{78}{64}$ | 0 | 0 | 0 |
| 7 | $\frac{6}{64}$ | $\frac{42}{64}$ | 1 | 1 | $1 * \frac{6}{64}$ |
| 8 | $\frac{12}{64}$ | $\frac{96}{64}$ | 2 | 4 | $4 * \frac{12}{64}$ |
| 9 | $\frac{8}{64}$ | $\frac{72}{64}$ | 3 | 9 | $9 * \frac{8}{64}$ |
| | | <u>6</u> | | | <u>$\frac{9}{2}$</u> |

Property #1

$$\mu = \mu_{\bar{x}}$$

Property #2

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

$$\mu_{\bar{x}} = \sum [\bar{x} P(\bar{x})] = \dots = 6$$

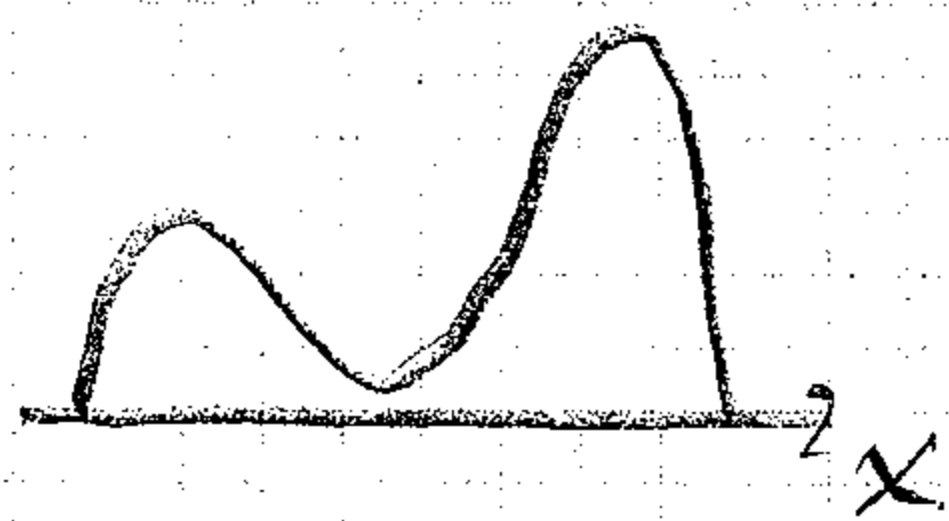
$$\sigma_{\bar{x}}^2 = \sum [(\bar{x} - \mu_{\bar{x}})^2 P(\bar{x})] = \frac{9}{2}$$

THE CENTRAL LIMIT THEOREM

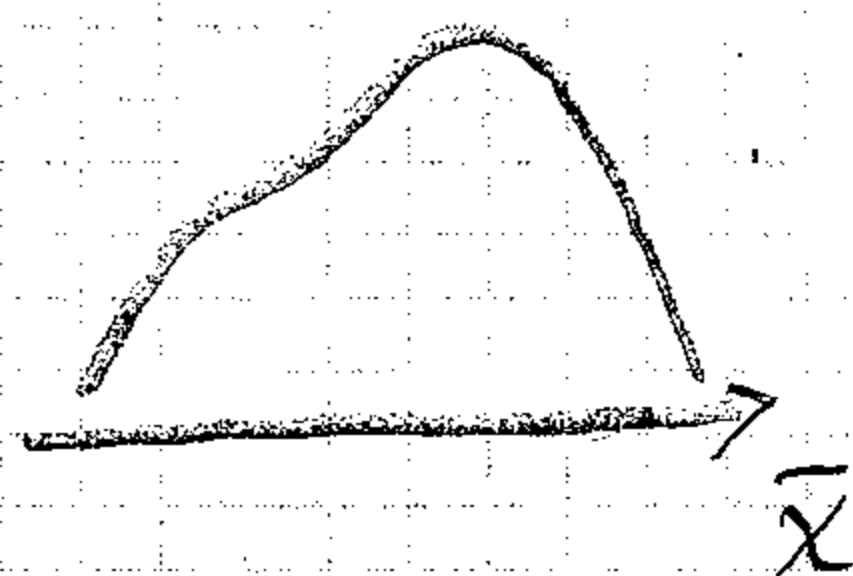
Let's say we take random samples of size n from a population with mean μ and standard deviation σ

when n is "sufficiently large", the sampling distribution of \bar{x} will be "approximately" normal with mean μ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. The larger the sample size, the better the normal approximation

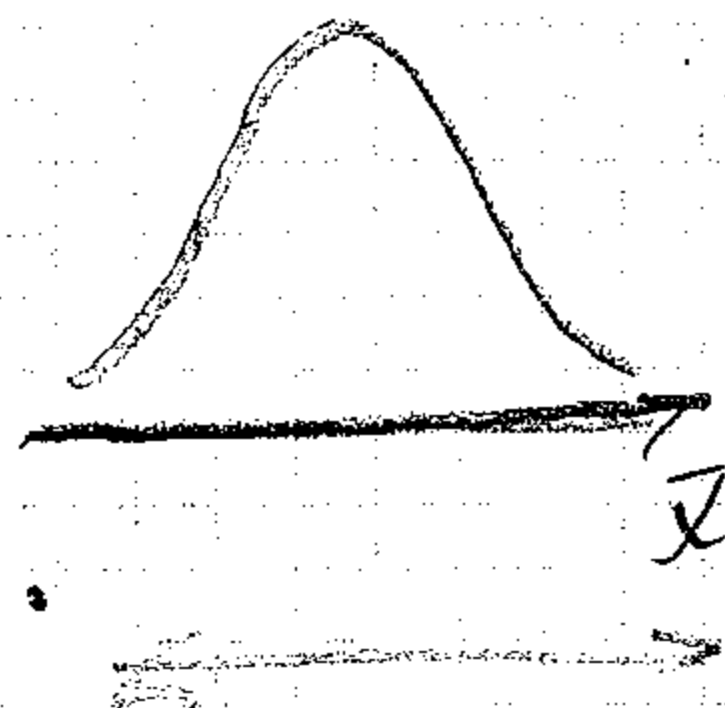
Original Population



Sampling Distribution of \bar{x} for $n = 3$



Sampling Distribution of \bar{x} for $n = 10$



Sampling Distribution of \bar{x} for $n = 30$

