

COMPLEX NUMBERS

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

$$x + 2 = 2 \Rightarrow x = 0$$

$$\mathbb{W} = \{0, 1, 2, 3, \dots\}$$

$$x + 4 = 2 \Rightarrow x = x = -2$$

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$$

$$2x = 3 \Rightarrow x = 3/2$$

$$\mathbb{Q} = \{x \mid x = \frac{a}{b}, a, b \in \mathbb{Z} \text{ and } b \neq 0\}$$

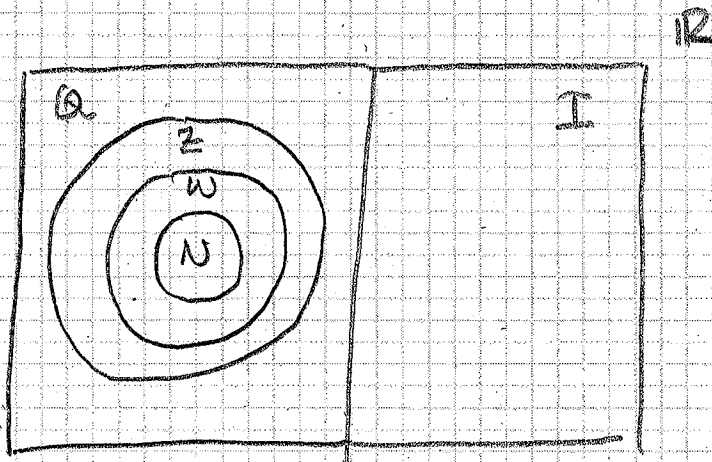
$$x^2 = 2 \Rightarrow x = \sqrt{2}$$

$\mathbb{I}$  = irrational numbers

$$\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$$

$$x^2 + 1 = 0 \Rightarrow x^2 = -1 \Rightarrow x = \sqrt{-1}$$

Imaginary unit  $i$       $i^2 = -1$



Complex Number

$$a + bi \quad a \in \mathbb{R}, b \in \mathbb{R}$$

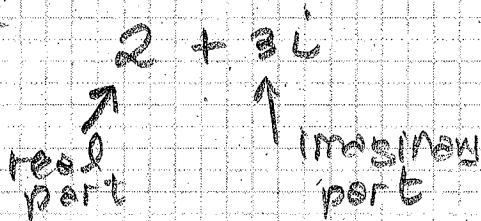
if  $a = 0$

$bi$  pure imaginary number

if  $b = 0$

$a$  real number

Example



$a+bi = c+di$  if and only if  $a=c$  and  $b=d$

$$(a+bi) + (c+di) = (a+c) + (b+d)i$$

$$(a+bi) - (c+di) = (a-c) + (b-d)i$$

$$\begin{aligned} (a+bi)(c+di) &= ac + adi + bi \cdot c + bc \cdot di \\ &= ac + (ad+bc)i + bdi^2 \\ &= (ac - bd) + (ad+bc)i \end{aligned}$$

$$\frac{2+3i}{4-5i} = \frac{(2+3i)(4+5i)}{(4-5i)(4+5i)}$$

$$= \frac{8+10i+12i+15i^2}{16-25i^2} = \frac{-7+22i}{41}$$

$$= -\frac{7}{41} + \frac{22}{41}i$$

$\frac{2}{3-\sqrt{2}}$   
 $(a+b)(a-b) = a^2 - b^2$   
 $a+bi$   
standard  
form

Powers of i

$i^{-8} = 1$
$i^{-7} = i$
$i^{-6} = -1$
$i^{-5} = -i$
$i^{-4} = 1$
$i^{-3} = i$
$i^{-2} = -1$
$i^{-1} = -i$
$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = i^2 \cdot i = -i$
$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$
$i^5 = i^4 \cdot i = 1 \cdot i = i$
$i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$
$i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$
$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$

$i^{-1} = \frac{1}{i}$  the conjugate of  $0+1i$  is  $0-1i = -i$

$$= \frac{1}{i} \cdot \frac{-i}{-i} = \frac{-i}{-i^2} = \frac{-i}{1} = -i$$

$$i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$$

$$i^{-3} = \frac{1}{i^3} = \frac{1}{i^2 \cdot i} = \frac{1}{-i} = -\frac{1}{i} = -(-i) = i$$

the powers of i are periodic and the period is 4

Ex:  $i^7 = i^3 = -i$

$$4 \overline{) \begin{array}{r} 17 \\ 4 \\ \hline 1 \end{array}}$$

Ex:  $i^{-25} = \frac{1}{i^{25}} = \frac{1}{i}$

$$= \frac{1}{i} \cdot \frac{-i}{-i} = \frac{-i}{-i^2} = \frac{-i}{1} = -i$$

$0+i$     $0-i$