

## Composition of Functions

The composition of two functions  $f$  and  $g$ , represented by  $f \circ g$  is defined by

$$(f \circ g)(x) = f(g(x))$$

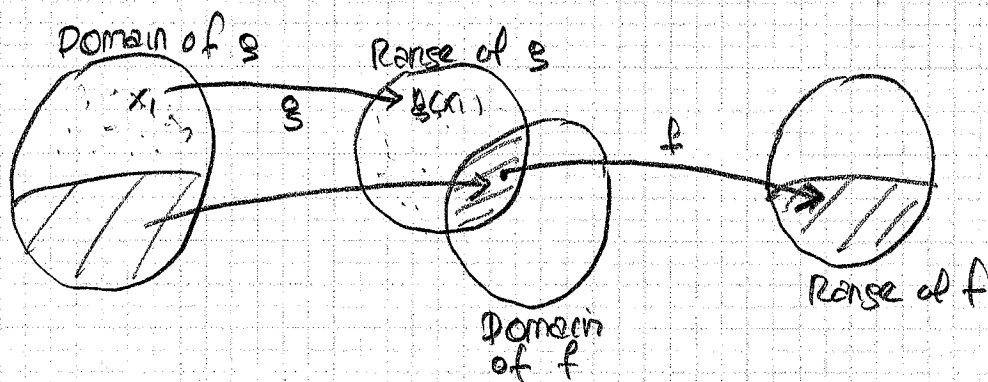
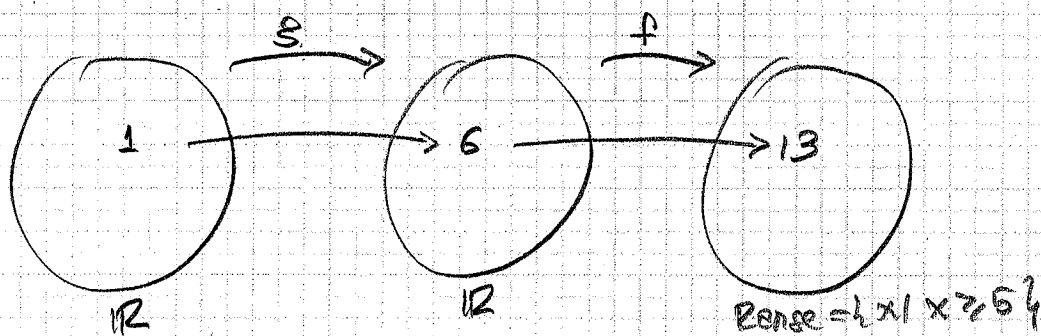
EX:  $f(x) = 2x + 1 \rightarrow \text{Dom}(f) = \mathbb{R} = (-\infty, +\infty)$

$g(x) = x^2 + 5 \Rightarrow \text{Dom}(g) = \mathbb{R} = (-\infty, +\infty)$

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 5) = 2(x^2 + 5) + 1$$

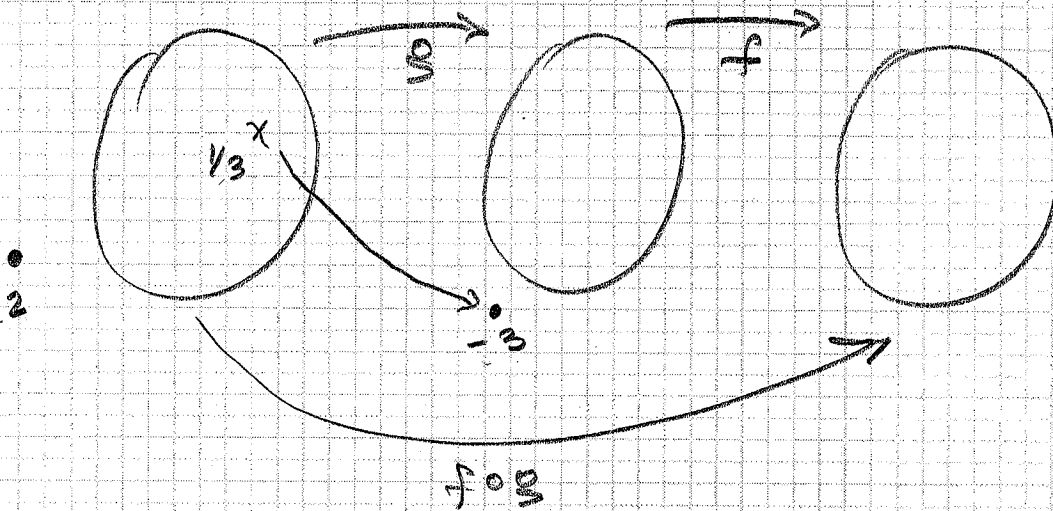
$$(f \circ g)(1) = 2(1^2 + 5) + 1 = 13$$

$$(f \circ g)(1) = f(g(1)) = f(6) = 13$$



Ex:  $f(x) = \frac{2}{x+3} \Rightarrow \text{Dom}(f) = \{x / x \neq -3\}$  Find  $\text{Dom}(f \circ g)$

$g(x) = \frac{5}{x-2} \Rightarrow \text{Dom}(g) = \{x / x \neq 2\}$



$(f \circ g)(2) = f(g(2)) = f\left(\frac{5}{2-2}\right) = f\left(\frac{5}{0}\right)$  *undef*

$(f \circ g)(-3) = f(g(-3)) = f\left(\frac{5}{-3-2}\right) = f(-1) = \frac{2}{-1+3} = \frac{2}{2} = 2$

$g(x) = \frac{5}{x-2} = -3 \Rightarrow 5 = -3(x-2)$

$\Rightarrow 5 = -3x + 6 \Rightarrow 3x = 6 - 5 = 1 \Rightarrow x = \frac{1}{3}$

$(f \circ g)\left(\frac{1}{3}\right) = f\left(g\left(\frac{1}{3}\right)\right) = f\left(\frac{5}{\frac{1}{3}-2}\right) = f(-3) = \frac{2}{-3+3}$  *undef.*

$\frac{1}{3} - \frac{2}{1} = \frac{1 \cdot 1 - 3 \cdot 2}{3} = \frac{1-6}{3} = \frac{-5}{3}$

$\frac{5}{-5/3} = \neq \left(-\frac{3}{5}\right) = -3$

$$\text{Dom}(f \circ g) = \{x \mid x \neq 2, \neq 3\}$$

$$f(x) = \frac{2}{x+3}$$

$$g(x) = \frac{5}{x-2} \Rightarrow \text{Dom}(g) = \{x \mid x \neq 2\}$$

$$\text{Dom}(f \circ g) = ?$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{5}{x-2}\right) = \frac{2}{\frac{5}{x-2} + 3} = \frac{2}{\frac{1 \cdot 5 + (x-2) \cdot 3}{x-2}}$$

$$= \frac{2}{\frac{5 + 3x - 6}{x-2}} = \frac{2}{\frac{3x-1}{x-2}} = 2 \cdot \frac{x-2}{3x-1} = \frac{2(x-2)}{3x-1}$$

$$3x-1=0 \Rightarrow 3x=1 \Rightarrow x = \frac{1}{3}$$