SIMPLE INTEREST

Deposit	$500	Principal	$P = 500$
Interest rate	4%		$r = .04$
time in years	5		$t = 5$
Interest earned			$I = ?$

$$\boxed{I = P \times r \times t} = 500 \times .04 \times 5 = 100$$

$$\begin{array}{r} 500 \\ 100 \\ \hline 600 \end{array}$$

Compound Interest

Present Value	P
Accumulated Value (Future Value)	A
number of times interest is compounded per year	n
interest rate as a decimal.	r
time in year	t

$$A = P \cdot \left(1 + \frac{r}{n}\right)^{n \cdot t}$$

Ex: $P = 500$

$r = .04$

$t = 5$

$n = 12$ (interest compounded monthly)

$A = ?$

$$A = 500\left(1 + \frac{.04}{12}\right)^{12*5} \approx 610.49$$

Continuous Compounding

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

if $n \to \infty$ $\quad A \to Pe^{rt}$

$$\boxed{A = Pe^{rt}}$$

$P = 500$
$r = .04$
$t = 5$

$$A = 500 * e^{.04 * 5} \approx 610.70$$

Ex: Interest rate needed to double an investment in 5 years if interest is compounded quaterly?

$n = 4$ $\quad 2\cancel{P} = \cancel{P}\left(1 + \frac{r}{n}\right)^{nt}$

$$2 = \left(1 + \frac{r}{4}\right)^{4 \cdot 5} \to 2 = \left(1 + \frac{r}{4}\right)^{20}$$

$$\Rightarrow \sqrt[20]{2} = 1 + \frac{r}{4} \Rightarrow \sqrt[20]{2} - 1 = \frac{r}{4} \Rightarrow$$

$$\Rightarrow \left(\sqrt[20]{2} - 1\right) \cdot 4 = r \Rightarrow r = .141 \quad (14.1\%)$$

Ex: Time needed to double an investment if the interest rate is 4% and interest is compounded continuously

$r = .04$

$$2P = Pe^{rt} \Rightarrow 2 = e^{.04t} \Rightarrow$$

$$\Rightarrow \ln 2 = \ln e^{.04t} \Rightarrow \ln 2 = .04t \cdot \ln e$$

$$\Rightarrow \ln 2 = .04t \Rightarrow \frac{\ln 2}{.04} = \boxed{t \approx 17.33}$$