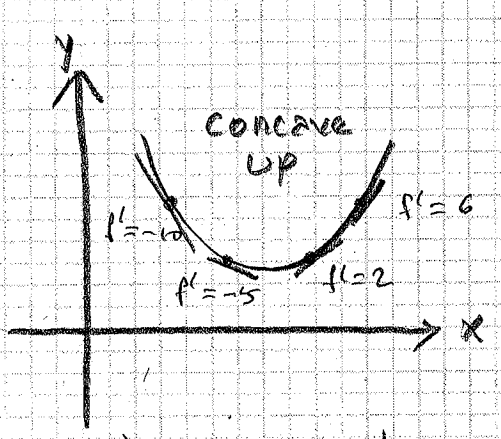
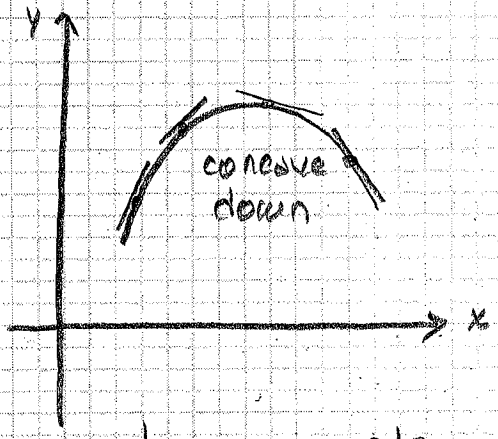
CONCAVITY AND INFLECTION POINTS



increasing slopes

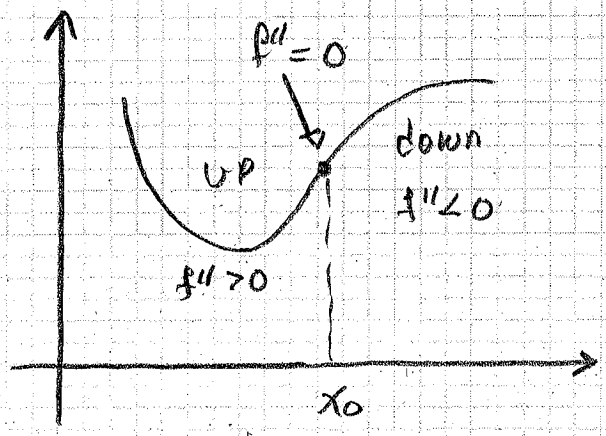
$f'' > 0$



decreasing slopes

$f'' < 0$

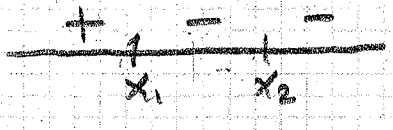
INFLECTION POINTS



If f is continuous on an open interval containing x_0 and f changes its concavity at x_0, we call $(x_0, f(x_0))$ an Inflection Point

TO FIND INFLECTION POINTS

1) Find the domain of f
2) Find $f'(x)$ and $f''(x)$
3) Find all the points in the domain of f where $f''(x)=0$ or $f''(x)$ is undefined
4) Plot those points on a line



Example $f(x) = x^{7/3} = \sqrt[3]{x^7}$

Domain $= (-\infty, \infty)$

$f'(x) = \frac{7}{3} x^{4/3}$

$f''(x) = \frac{28}{9} x^{1/3} = \frac{28}{9} \sqrt[3]{x}$

$f''(x) = 0$

$\frac{28\sqrt[3]{x}}{9} = 0$

$x = 0$

$f''(x)$ undefined
no values

concave down | concave up

$-$ | $+$ $\quad f''$

$\uparrow$ 0 $\uparrow$

f has an IP at $x=0$ $\quad y=0$ $\quad (0,0)$

Example: $f(x) = x^{7/5} = \sqrt[5]{x^7}$

Domain $= (-\infty, \infty)$

$f'(x) = \frac{7}{5} x^{2/5}$

$f''(x) = \frac{14}{25} x^{-3/5} = \frac{14}{25\sqrt[5]{x^3}}$

$f''(x) = 0$

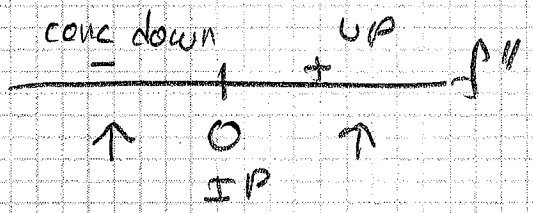
$\frac{14}{25\sqrt[5]{x^3}} = 0$

NO solutions

$f''(x)$ undf

$\frac{14}{25\sqrt[5]{x^3}}$ undf $\Rightarrow$

$\Rightarrow x = 0$



ex: $f(x) = x^4$

Domain = $(-\infty, \infty)$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''(x) = 0$ or $f''(x)$ undf

NO values

$12x^2 = 0$

$\boxed{x = 0}$