Confidence Interval for the Ratio of two Variances.

Derivation of the formula

The sampling distribution of $V_1 = \frac{(n_1-1)s_1^2}{\sigma_1^2}$ follows a Chi-Square distribution with n_1-1 degrees of freedom, if the population that it comes from is normal.

Similarly, $V_2 = \frac{(n_2-1)s_1^2}{\sigma_2^2}$, follows a Chi-square distribution with n_2-1 degrees of freedom.

and

$$F = \frac{\frac{V_1}{n_1-1}}{\frac{V_2}{n_2-1}}$$

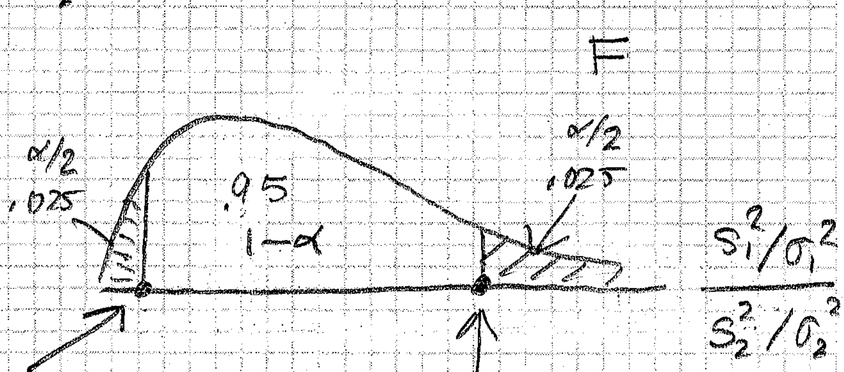
follows an F distribution with n_1-1 degrees of freedom for the numerator, and n_2-1 degrees of freedom for the denominator

$$F = \frac{\frac{\cancel{(n_1-1)}s_1^2}{\sigma_1^2}\Big/\cancel{(n_1-1)}}{\frac{\cancel{(n_2-1)}s_2^2}{\sigma_2^2}\Big/\cancel{(n_2-1)}} = \frac{\frac{s_1^2}{\sigma_1^2}}{\frac{s_2^2}{\sigma_2^2}} = \frac{\sigma_2^2}{\sigma_1^2} * \frac{s_1^2}{s_2^2}$$

95% C.I

$\alpha = 1 - .95 = .05$

$\alpha/2 = .025$



$F_{1-\alpha/2,\, n_1-1,\, n_2-1}$ $F_{\alpha/2,\, n_1-1,\, n_2-1}$

$$P\left[F_{1-\alpha/2;\, n_1-1,\, n_2-1} \le \frac{\sigma_2^2}{\sigma_1^2} * \frac{s_1^2}{s_2^2} \le F_{\alpha/2,\, n_1-1,\, n_2-1}\right] = 1-\alpha$$

$$3 \le 5 \le 7 \Rightarrow \frac{1}{3} \ge \frac{1}{5} \ge \frac{1}{7}$$

$$P\left[\frac{1}{F_{1-\alpha/2,\, n_1-1,\, n_2-1}} \ge \frac{\sigma_1^2}{\sigma_2^2} * \frac{s_2^2}{s_1^2} \ge \frac{1}{F_{\alpha/2,\, n_1-1,\, n_2-1}}\right] = 1-\alpha$$

$$P\left[\frac{1}{F_{\alpha/2,\, n_1-1,\, n_2-1}} \le \frac{\sigma_1^2}{\sigma_2^2} * \frac{s_2^2}{s_1^2} \le \frac{1}{F_{1-\alpha/2,\, n_1-1,\, n_2-1}}\right] = 1-\alpha$$

$$P\left[\frac{s_1^2}{s_2^2}\frac{1}{F_{\alpha/2,\, n_1-1,\, n_2-1}} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2}\frac{1}{F_{1-\alpha/2,\, n_1-1,\, n_2-1}}\right] = 1-\alpha$$

Property: $\frac{1}{F_{1-\alpha/2,\, n_1-1,\, n_2-1}} = F_{\alpha/2,\, n_2-1,\, n_1-1}$

$$P\left[\frac{s_1^2}{s_2^2}\frac{1}{F_{\alpha/2,\, n_1-1,\, n_2-1}} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} * F_{\alpha/2,\, n_2-1,\, n_1-1}\right] = 1-\alpha$$

$$\boxed{\left(\frac{s_1^2}{s_2^2} * \frac{1}{F_{\alpha/2,\, n_1-1,\, n_2-1}} \,,\; \frac{s_1^2}{s_2^2} * F_{\alpha/2,\, n_2-1,\, n_1-1}\right)}$$