

I ended up covering  $Z$  and  $t$  intervals in this video

## TWO SAMPLE Z INTERVALS

### Paired Samples versus Independent Samples

#### Paired

- Every observation in one of the samples has an observation in the other sample that is directly related to it

#### Independent

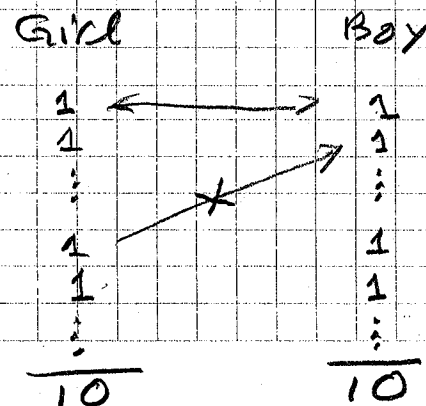
- That is not the case

Example: Let's say we want to compare the SAT scores of girls and boys. We will do this in two different ways

First: We take 10 boys at random from different schools and 10 girls, also at random, from different schools. We give them an SAT test, calculate the mean scores and compare them.

#### Second

School A, GPA 3.5  
 School A, GPA 3.0  
 ⋮  
 School B, GPA 3.5  
 School B, GPA 3.0



Example: Let's say we want to compare the effectiveness of medicines A and B to lower blood pressure.

So, we take a random sample of 50 patients suffering from HBP, administer medicine A to all patients and later check the blood pressure.

Then, wait enough time to allow any residual effect to go away and administer medicine B to the same patients

Paired?

Independent?

Before (A)

After (B)

Joe ←————→ Joe

Jackie ←————→ Jackie

Elizabeth ∴

Elizabeth

∴

∴

Paired

Independent

Both samples have the same number of observations

they may or may not

### ONE SAMPLE CONFIDENCE INTERVAL FOR THE MEAN

$\bar{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}} \text{ when } n \geq 30$	$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \text{ when } \sigma \text{ is known}$
$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \text{ when } n < 30$	$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \text{ when } \sigma \text{ is unknown}$ <p>where</p> <p>df=n-1</p>

### CONFIDENCE INTERVALS FOR INDEPENDENT SAMPLES

#### TWO SAMPLE INTERVAL FOR THE DIFFERENCE BETWEEN TWO POPULATION MEANS

<p>If both samples are large</p> $\bar{x}_1 - \bar{x}_2 \pm Z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	<p>If <math>\sigma_1</math> and <math>\sigma_2</math> are known</p> $\bar{x}_1 - \bar{x}_2 \pm Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
<p>if samples are small but we can assume that the variances are equal</p> $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$ <p>where:</p> $S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ <p>And <math>df = n_1 + n_2 - 2</math></p> <p>Both Samples must be approximately normally distributed with equal population variances and samples must be independent</p>	<p>If <math>\sigma_1</math> and <math>\sigma_2</math> are unknown</p> $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ <p>where</p> $df = \frac{\left( \frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} \right)^2}{\frac{1}{n_1 - 1} \left( \frac{S_1^2}{n_1} \right)^2 + \frac{1}{n_2 - 1} \left( \frac{S_2^2}{n_2} \right)^2}$ <p>not assuming the equality of the variances.</p> <p>For most applications a sample size of <math>n \geq 30</math> is adequate. If the populations distribution is approximately normal, smaller samples may be used. If the distribution is highly skewed, sizes of <math>n \geq 50</math> are recommended.</p>

Example: Independent random samples from two populations are selected.

Sample 1

$$n_1 = 50$$

$$\bar{x}_1 = 17$$

$$s_1 = 2$$

Sample 2

$$n_2 = 60$$

$$\bar{x}_2 = 16$$

$$s_2 = 4$$

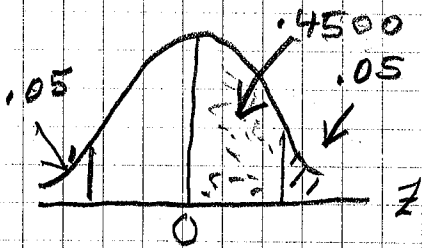
Calculate a 90% confidence interval for  $\mu_1 - \mu_2$ .

Solution (by the sample sizes)

$$n_1 = 50 > 30$$

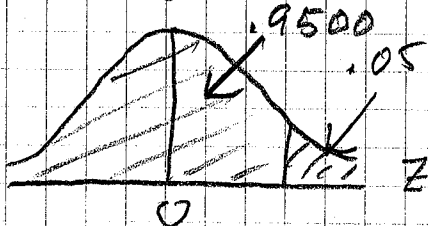
$$n_2 = 60 > 30$$

$$C.I. = \bar{x}_1 - \bar{x}_2 \pm Z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$



$$C.I. = 90\% \Rightarrow \alpha = 1 - 0.90 = 0.10$$

$$\alpha/2 = 0.05 \quad Z_{\alpha/2} = 1.645$$



$$17 - 16 \pm 1.645 \sqrt{\frac{2^2}{50} + \frac{4^2}{60}}$$

$$= 1 \pm 0.969 = (.031, 1.969)$$

Solution 2

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$df = \frac{\left( \frac{2^2}{50} + \frac{4^2}{60} \right)^2}{\frac{1}{50-1} \left( \frac{2^2}{50} \right)^2 + \frac{1}{60-1} \left( \frac{4^2}{60} \right)^2} = 42.34$$

= 42 which we round down to provide a larger t-value and a more conservative (wider) interval estimate

$$t_{\alpha/2} = 1.682$$

$$1 \pm .990 = (.01, 1.99)$$