CONFIDENCE INTERVALS FOR THE MEAN WITH PAIRED SAMPLES

Pair	Var X_1	Var X_2	$d = X_1 - X_2$	
1	7	4	$7-4=3$	$= d_1$
2	9	5	$9-5=4$	$= d_2$
3	5	4	$5-4=1$	$\vdots$
4	8	3	$8-3=5$	$\vdots$
5	6	4	$6-4=2$	
6	4	3	$4-3=1$	$= d_6$

$$\bar{d} = \frac{\Sigma d}{n_d} = \frac{3+4+1+5+2+1}{6} \approx 2.67$$

d	$d-\bar{d}$	$(d-\bar{d})^2$
3	$3-2.67$	.1089
4	$4-2.67$	1.7689
1	$1-2.67$	2.7889
5	$5-2.67$	5.4289
2	$2-2.67$	.4489
1	$1-2.67$	2.7889

$$S_d = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n_d - 1}} \approx 1.63$$

n_d
$\bar{d}$
S_d

$$\bar{d} \pm t_{\alpha/2} \frac{S_d}{\sqrt{n_d}} = 2.67 \pm t_{\alpha/2} \frac{1.63}{\sqrt{6}}$$

95% C.I. $\Rightarrow \alpha = 1 - .95 = .05 \Rightarrow \alpha/2 = .025$

$df = n_d - 1 = 5 \qquad t_{\alpha/2} = 2.571$

C.I.: $2.67 \pm 2.571 \times \dfrac{1.63}{\sqrt{6}} = 2.67 \pm 1.71 = (.96,\ 4.38)$

ONE SAMPLE CONFIDENCE INTERVAL FOR THE MEAN

$\overline{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}$ when $n \geq 30$	$\overline{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ when σ is known
$\overline{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ when n<30 where df=n-1	$\overline{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ when σ is unknown where df=n-1

CONFIDENCE INTERVALS FOR PAIRED (MATCHED) DIFFERENCES

If the number of differences is large $\overline{d} \pm Z_{\alpha/2} \frac{S_d}{n_d}$	If σ_d is known $\overline{d} \pm Z_{\alpha/2} \frac{\sigma_d}{n_d}$
if the number of differences is small $\overline{d} \pm t_{\alpha/2} \frac{S_d}{\sqrt{n_d}}$ where: df = $n_d - 1$ The population of differences must have a distribution that is approximately normal.	If σ_d is unknown $\overline{d} \pm t_{\alpha/2} \frac{S_d}{\sqrt{n_d}}$ where: df = $n_d - 1$ For most applications, a number of differences greater than or equal than 30 is adequate. If the population of differences has a distribution that is approximately normal, smaller samples may be used. If the distribution of differences is highly skewed, sizes of n$\geq$50 are recommended.