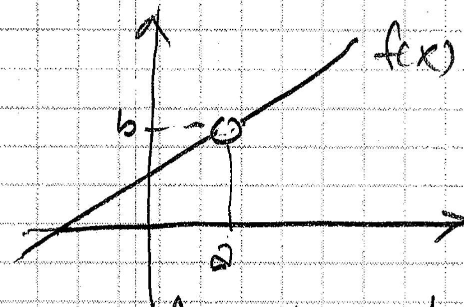
CONTINUITY

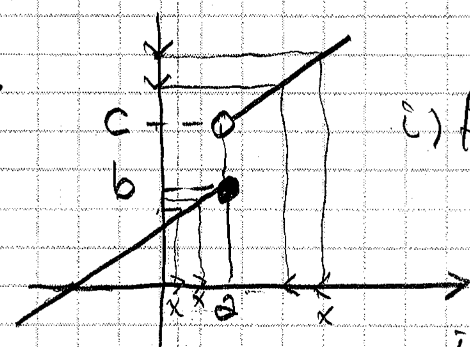
A funcion $f(x)$ is continuous at $x=a$ if:

i) $f(a)$ exists



$f(a)$ does not exist
$\therefore$ $f(x)$ is not continuous at $x=a$

ii) $\lim_{x \to a} f(x)$ exists

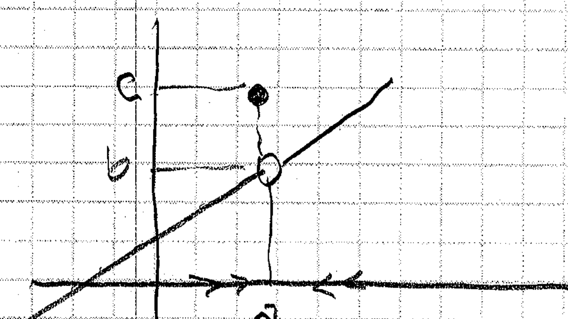


i) $f(a)=b$ ✓

ii) ✗

$$\left.\begin{array}{l} \lim_{x \to a^-} f(x) = b \\ \lim_{x \to a^+} f(x) = c \end{array}\right\} \lim_{x \to a} f(x) \text{ d.n.e.}$$

iii) $\lim_{x \to a} f(x) = f(a)$



i) $f(a)=c$ ✓

ii) $\lim_{x \to a} f(x) = b$ ✓

iii) $\lim_{x \to a} f(x) \neq f(a)$ ✗

THREE IMPORTANT RESULTS ABOUT CONTINUITY

$$\boxed{\text{Polynomials are continuous for every value of } x}$$

$f(x) = 3x^2 - 5x + 7$

the exponents must be non-negative integers

$f(x) = \frac{3}{x} - 5x + 7$ Not a polynomial

$3 \cdot x^{-1}$

$f(x) = 3\sqrt{x} - 5x + 7$ Not " "

$3x^{1/2}$

$f(x) = \sqrt{3}x^2 - 5x + \frac{1}{2}$ It is a poly...

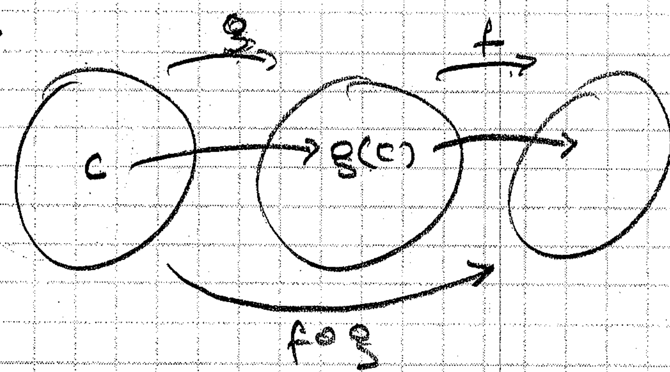
$3^{1/2}$ 2^{-1}

Rational functions, $R(x) = \frac{\text{polynomial}}{\text{polynomial}}$, are continuous everywhere except wherever the denominator is zero.

$R(x) = \frac{2x+3}{x-2}$ is it continuous at $x=1$? Yes

" " " $x=2$? No

1) If the function g is continuous at c and the function f is continuous at g(c) then the composition f∘g is continuous at c



Ex: $g(x) = \frac{1}{x}$ not continuous at $x=0$

$f(x) = \frac{2x+3}{x-2}$ not continuous at $x=2$

Is the composition f∘g continuous at $x=1$?

Yes, because g(x) is continuous at $x=1$

$g(1) = \frac{2\cdot1+3}{1-2} = \frac{5}{-1} = -5$

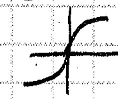
2) If the function g is continuous everywhere and the function f is continuous everywhere then, the composition f∘g is continuous everywhere

Ex: $g(x) = x^2$ $f(x) = \frac{x+1}{x^2+1}$

Exercises: Find the values of x, if any, at which f is not continuous

1) $f(x) = 3x^5 - 2x + 7$ None

2) $f(x) = \sqrt[3]{x-1}$ $y = x-1$ poly $\therefore$ cont

$y = \sqrt[3]{x}$  None

3) $f(x) = \dfrac{4x-2}{x+3}$ $x = -3$

4) $f(x) = \dfrac{2x}{x^2+x}$ $x^2+x=0 \Rightarrow x(x+1)=0$

$x=0$ $x=-1$

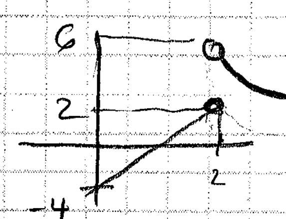
5) $f(x) = \dfrac{2}{x} - \dfrac{x+1}{x^2-1} = \dfrac{2}{x} - \dfrac{x+1}{(x+1)(x-1)}$ $x=0$, $x=-1$, $x=1$

6) $f(x) = \dfrac{x^2+5x+6}{|x|+1}$ None

7) $f(x) = \left| 2 - \dfrac{5}{x^4+x^2} \right|$ $x^4+x^2 = x^2(x^2+1) = 0$

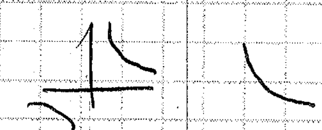
$x=0$

8) $f(x) = \begin{cases} 3x-4 & x \le 2 \\ 5 + \dfrac{2}{x} & x > 2 \end{cases}$



$y = 3x-4$ cont for $x < 2$

$y = 5 + \dfrac{2}{x}$ cont for $x > 2$

$f(x) = \dfrac{1}{x}$ 

$\boxed{\text{At } x = 2}$

i) $f(2) = 3 \cdot 2 - 4 = 6 - 4 = 2$ ✓

ii) $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} (3x-4) = 2$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} \left(5 + \dfrac{2}{x}\right) = 5 + 1 = 6$