COUNTING METHODS

How many different 4-letter radio station call letters can be made if the first letter must be W and no letter may be repeated?

How many committees of 3 people can be made from 10 people?

In how many ways can we elect a President, a Vice President and a Secretary from 10 people?

Combinatorics

By Sistematic Listing using { Tables / Trees

or

Using Formulas

Example: To get dressed you must choose shirt, pants and shoes. If you have 3 shirts 2 pairs of pants and 4 pairs of shoes, in how many ways can you get dressed?

$$\underset{\text{shirts}}{3} \times \underset{\text{pants}}{2} \times \underset{\text{shoes}}{4} = 3 \times 2 \times 4 = 24$$

The Fundamental Counting Principle

Task: Task 1 Task 2 Task 3

$n_1 \times n_2 \times n_3$

Example: Radco Staten Letters

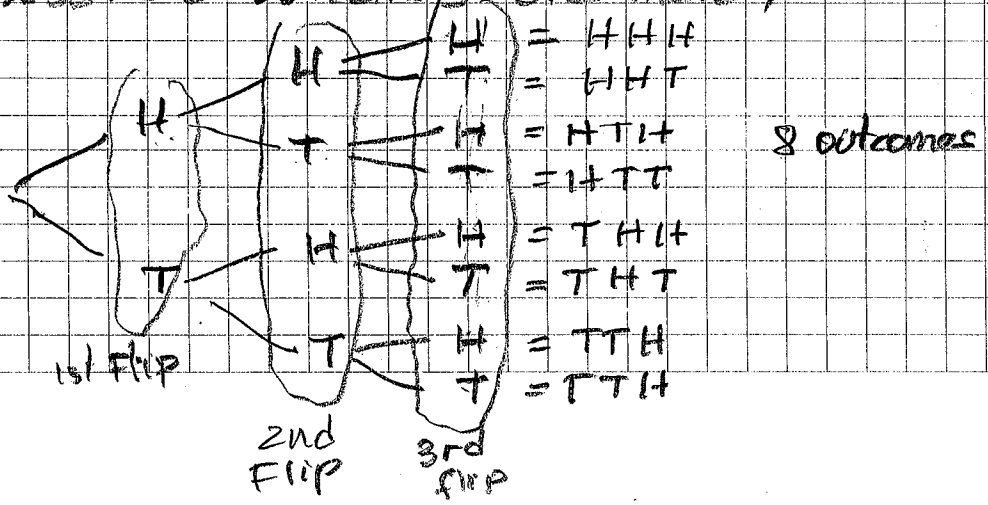
$\frac{1}{W} \times \frac{25}{} \times \frac{24}{} \times \frac{23}{} = 13,800$

Example: We are flipping a coin twice. How many possible outcomes are there?

First Flip \ 2nd Flip	H	T
H	HH	HT
T	TH	TT

4 outcomes

Example: We are flipping a coin 3 times. How many possible outcomes are there?



8 outcomes

Permutations of n elements taken r at a time (where order matters and without repetition)

$$nPr = \frac{n!}{(n-r)!}$$

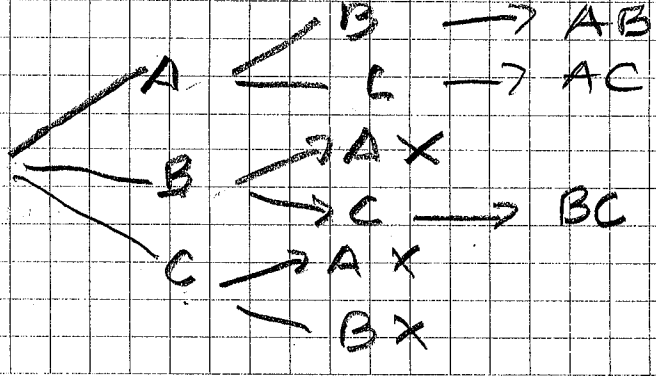
$n = 6$
$r = 4$

$$6P_4 = \frac{6!}{(6-4)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} = 360$$

Combinations of n elements taken r at a time (where order does NOT matter and without repetitions)

$$nCr = \frac{n!}{r!(n-r)!}$$

Example: How many different committees of 2 people can we make from 3 people?



3 committees

$$3C2 = \frac{3 \cdot 2!}{2!(3-2)!} = 3$$

FORMULAS: Permutations and Combinations

FACTORIALS

$3! = 3 \cdot 2 \cdot 1$

$4! = 4 \cdot 3 \cdot 2 \cdot 1$

$\vdots$

$n! = n(n-1) \cdots 1$

Define $0! = 1$

Example: In how many ways can we seat 4 people on the 4 chairs of your dining room table?

$$\underset{ch\,1}{4} \times \underset{ch\,2}{3} \times \underset{ch\,3}{2} \times \underset{ch\,4}{1} = 4 \cdot 3 \cdot 2 \cdot 1 = 4!$$

"arrange" "order matters"

Example: Now you have 6 people and only 4 chairs. In how many ways can we choose 4 people from 6, if order matters and noone can be repeated?

$$\underset{ch\,1}{6} \times \underset{ch\,2}{5} \times \underset{ch\,3}{4} \times \underset{ch\,4}{3} = 360$$