

DEFINITE INTEGRALS BY SUBSTITUTION

Example:  $I = \int_0^3 x^2 (x^3 + 1)^5 dx$

Method 1 Evaluate the indefinite integral

$$\int x^2 (x^3 + 1)^5 dx = \frac{1}{3} \int (x^3 + 1)^5 \underbrace{3 \cdot x^2 dx}$$

$$\left. \begin{array}{l} u = x^3 + 1 \\ du = 3x^2 dx \end{array} \right\} = \frac{1}{3} \int u^5 du = \frac{1}{3} \frac{u^6}{6} + C$$

$$= \frac{1}{18} (x^3 + 1)^6 + C$$

now, we evaluate the definite integral

$$I = \left. \frac{1}{18} (x^3 + 1)^6 \right|_0^3 = \frac{1}{18} (3^3 + 1)^6 - \frac{1}{18} (0^3 + 1)^6$$

$$= 26,771,683.56$$

Method 2 changing the limits of integration

$$I = \int_0^3 x^2 (x^3 + 1)^5 dx = \frac{1}{3} \int_0^3 (x^3 + 1)^5 \cdot 3 \cdot x^2 dx$$

$$\left. \begin{array}{l} u = x^3 + 1 \\ du = 3x^2 dx \end{array} \right\} \begin{array}{l} \text{when } x=0, u=0^3+1=1 \\ \text{when } x=3, u=3^3+1=28 \end{array}$$

$$I = \frac{1}{3} \int_1^{28} u^5 du = \frac{1}{3} \frac{u^6}{6} \Big|_1^{28} = \frac{1}{18} [u^6]_1^{28}$$

$$= \frac{1}{18} [28^6 - 1^6] = 26,771,683.56$$