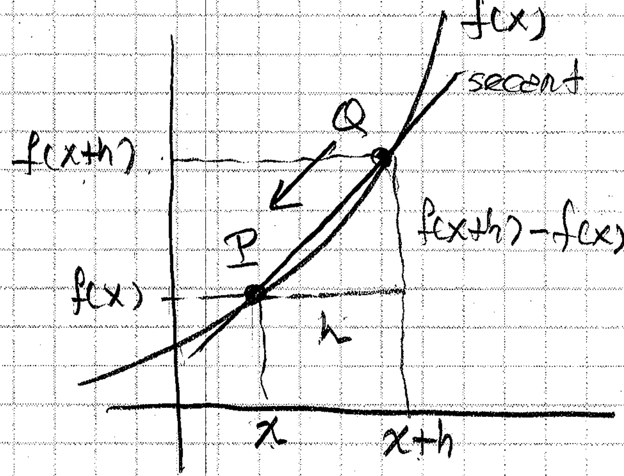
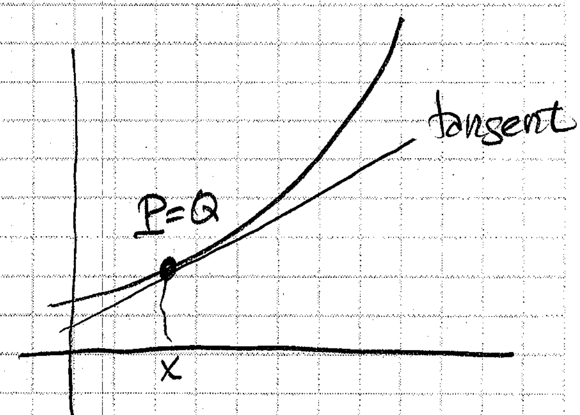
DERIVATIVES



$\frac{f(x+h)-f(x)}{h} =$

= slope of the secant line thru P & Q

= average rate of change of f(x) in [x, x+h]



$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

= slope of the tangent line to f(x) at x

= instantaneous rate of change of f(x) at x

Calculate the derivative of the following functions using the definition of derivative as a limit

1) $f(x) = x^2$

$f(x+h) = (x+h)^2$

$= x^2 + 2xh + h^2$

not $x^2 + h$

$(a+b)^2 \cancel{=} a^2 + b^2$

$(a+b)^2 = (a+b)(a+b) =$

$a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{2xh + h^2} \to 0}{\cancel{h} \to 0} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x+h) = 2x$$ done

Later: $f(x) = x^n \qquad f'(x) = nx^{n-1}$

$f(x) = x^2 \qquad f'(x) = 2x$

2) $f(x) = \frac{1}{x} \qquad f'(x) = ? = \frac{dy}{dx}$

$$f(x+h) = \frac{1}{x+h} \quad \text{not} \quad \frac{1}{x} + h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cancel{\frac{1}{x+h} - \frac{1}{x}} \to 0}{\cancel{h} \to 0}$$

$$= \lim_{h \to 0} \frac{\frac{x - (x+h)}{(x+h)x}}{h} = \lim_{h \to 0} \frac{\frac{-h}{(x+h)x}}{h}$$

$$= \lim_{h \to 0} \left[\frac{-\cancel{h}}{(x+h)x} \cdot \frac{1}{\cancel{h}} \right] = \lim_{h \to 0} \frac{-1}{(x+h) \cdot x} =$$

$$= \frac{-1}{x^2} = -x^{-2}$$

$f(x) = x^n \qquad f'(x) = nx^{n-1}$

$f(x) = x^{-1} \qquad f'(x) = -1 \cdot x^{-2} = -x^{-2}$

3) $f(x)=\sqrt{x}$ $\quad f'(x)=?$

$f(x+h)=\sqrt{x+h}$ not $\sqrt{x}+h$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\overbrace{\sqrt{x+h}-\sqrt{x}}^{\to 0}}{\underbrace{h}_{\to 0}}=$$

$$=\lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})}{h}\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}=$$

$$=\lim_{h\to 0}\frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}=\frac{1}{2x^{1/2}}=\frac{1}{2}x^{-1/2}$$

$f(x)=\sqrt{x}=x^{1/2}$ $\quad f'(x)=?$

$f(x)=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-1/2}$