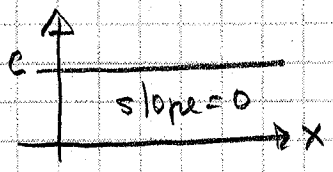
DERIVATIVES PART II

1) $f(x) = c \qquad f'(x) = 0 \qquad \frac{d(c)}{dx} = 0$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h}$$

$$= \lim_{h \to 0} (0) = 0$$

2) Power Rule

$f(x) = x^n \qquad f'(x) = n x^{n-1} \qquad \frac{d}{dx}[x^n] = n x^{n-1}$

3) $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$

$(c \cdot f)' = c \cdot f'$

4) $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$

$(f+g)' = f' + g'$

$(f-g)' = f' - g'$

EXERCISES Differentiate the given functions

1) $f(x) = x^{-3}$

$f'(x) = (-3)x^{-3-1} = -3x^{-4}$

2) $f(x) = 3$

$f'(x) = 0$

$(x^n)' = nx^{n-1}$

3) $f(x) = 2 \cdot x^3$

$(c \cdot f)' = c \cdot f'$

$f(x) = 2 \cdot 3 \cdot x^2 = 6x^2$

4) $f(x) = \sqrt{3x^3} = \sqrt{3} \cdot \sqrt{x^3} = \sqrt{3}\, x^{3/2}$

$f'(x) = \sqrt{3} \cdot \frac{3}{2} x^{\frac{3}{2}-1}$

$= \frac{3\sqrt{3}}{2} x^{1/2}$

$\sqrt[n]{x^m} = x^{m/n}$

$\frac{1}{x^n} = x^{-n}$

5) $f(x) = \frac{5}{\sqrt{x^5}} = \frac{5}{x^{5/2}} = 5 \cdot x^{-5/2}$

$f'(x) = 5\left(-\frac{5}{2}\right) \cdot x^{-5/2-1} = -\frac{25}{2} x^{-7/2}$

6) $f(x) = \frac{x^4 - 3x^3}{x^2} = \frac{x^4}{x^2} - \frac{3x^3}{x^2} = x^2 - 3x$

$f'(x) = (x^2)' - (3x)'$

$= 2x - 3 \cdot 1$

$= 2x - 3$

$(f+g)' = f' + g'$

$(f-g)' = f' - g'$

$(x)' = (x^1)' =$

$= 1 \cdot x^0 = 1$

PRODUCT RULE

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$$

$$(f \cdot g)' = g \cdot f' + g' \cdot f$$

QUOTIENT RULE

$$\left[\frac{f}{g}\right]' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)] \cdot g(x) - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$$

EX: $f(x) = (x-4)(1-3x)$

Calculate $f'(x)$ using the product rule

$f(x) = x - 3x^2 - 4 + 12x = -3x^2 + 13x - 4$

$f'(x) = -6x + 13$

$f'(x) = (x-4)'(1-3x) + (x-4) \cdot (1-3x)'$

$= 1(1-3x) + (x-4)(0-3)$

$= 1 - 3x + (x-4)(-3)$

$= 1 - 3x - 3x + 12 = 1 - 6x + 12$

$= -6x + 13$

EX: $f(x) = \dfrac{x^3+2}{1-x^2}$ $\quad f'(x)$

$\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$

$$f'(x) = \frac{(x^3+2)'\cdot(1-x^2) - x^3\cdot(1-x^2)'}{(1-x^2)^2}$$

$$= \frac{3x^2(1-x^2) - x^3(-2x)}{(1-x^2)^2} \quad \square$$

HIGHER ORDER DERIVATIVES

$f'(x)$ first derivative

$f''(x) = [f'(x)]'$