

## DERIVATIVES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

$$\frac{d}{dx} (\ln x) = \frac{1}{x} \quad x > 0$$

$$\frac{d}{dx} (\log_b x) = \frac{1}{x \cdot \ln b} \quad x > 0$$

$$\frac{d}{dx} (e^x) = e^x$$

$$\frac{d}{dx} (b^x) = b^x \cdot \ln b$$

Examples Find  $dy/dx$

1)  $y = \ln(2x)$  "Chain Rule"

$$\frac{d}{dx} (\ln(2x)) = \frac{1}{2x} \cdot (2x)' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$$

2)  $y = \ln|x|$  for  $x \neq 0$

If  $x > 0$  then  $|x| = x$

If  $x < 0$  then  $|x| = -x$       $|+2| = -(-2) = 2$

If  $x > 0$       $y = \ln|x| = \ln x$       $\frac{dy}{dx} = \frac{1}{x}$

If  $x < 0$       $y = \ln|x| = \ln(-x)$

$$\frac{dy}{dx} = \frac{1}{-x} (-1) = \frac{1}{x}$$

$$3) \quad y = \sqrt{\ln x} = (\ln x)^{1/2}$$

$$(x^n)' = nx^{n-1}$$

$$\frac{dy}{dx} = \frac{1}{2} (\ln x)^{-1/2} (\ln x)'$$

$$= \frac{\frac{1}{x}}{2 (\ln x)^{1/2}}$$

$$= \frac{1}{2x \sqrt{\ln x}}$$

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{d}{dx} x^{-n} = \frac{d}{dx} \frac{1}{x^n}$$

$$\frac{\frac{1}{x}}{\frac{1}{2}} = \frac{1}{2x}$$

$$4) \quad y = \ln(\ln(\ln x))$$

$$\frac{dy}{dx} = \frac{1}{\ln(\ln x)} \cdot (\ln(\ln x))'$$

$$= \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} (\ln x)'$$

$$= \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$5) \quad y = \frac{\log x}{3 + \log(2x)}$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{x \ln 10} \cdot (3 + \log(2x)) - \log x \cdot \left(\frac{1}{2x \cdot \ln 10} \cdot 2\right)}{(3 + \log(2x))^2}$$

$$6) \quad y = e^{3x}$$

$$\frac{dy}{dx} = e^{3x} (3x)' = 3e^{3x}$$

$$7) \quad y = \pi^{x-x^2}$$

$$(b^x)' = b^x \cdot \ln b$$

$$b = \pi$$

$$\begin{aligned} \frac{dy}{dx} &= \pi^{x-x^2} \cdot \ln \pi \cdot (x-x^2)' \\ &= \pi^{x-x^2} \cdot \ln \pi \cdot (1-2x) \end{aligned}$$