DERIVATIVES OF TRIGONOMETRIC AND INVERSE TRIGONOMETRIC FUNCTIONS

$(\sin x)' = \cos x$ $(\cos x)' = -\sin x$ $(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$ $(\sec x)' = \sec x \cdot \tan x$ $(\csc x)' = -\csc x \cdot \cot x$

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\tan^{-1} x)' = \frac{1}{1+x^2} \qquad (\cot^{-1} x)' = -\frac{1}{1+x^2}$$

$$(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}} \qquad (\csc^{-1} x)' = -\frac{1}{|x|\sqrt{x^2-1}}$$

Exercises: Find $y'(x)$

1) $y = \sin\left(\frac{2x}{x+1}\right)$ "chain rule" $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$$y' = \cos\left(\frac{2x}{x+1}\right) \cdot \left(\frac{2(x+1) - 2x}{(x+1)^2}\right)$$

2) $y = \sqrt[3]{\sec(5x)} = \left[\sec(5x)\right]^{1/3}$

$$y' = \frac{1}{3}\left[\sec(5x)\right]^{-2/3} \cdot \sec(5x)\tan(5x) \cdot 5$$

3) $y = \tan^3(\cos(3x)) = [\tan(\cos(3x))]^3$

$y' = 3[\tan(\cos(3x))]^2 \cdot \sec^2(\cos(3x)) \cdot (-\sin(3x)) \cdot 3$

4) $y = \sin^{-1}(\sqrt{x})$ for $x \geq 0$

$\sin^{-1}x \neq (\sin x)^{-1}$ $\sin^{-1}x = \arcsin x$

$y = \sin^{-1}(x^{1/2})$

$y' = \frac{1}{\sqrt{1-(x^{1/2})^2}} \cdot \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$

$\left[(\sqrt{x})^2 = \sqrt{x^2} = |x| \right.$
$\left. \text{if } x \geq 0 \quad \sqrt{x^2} = x \right]$

$= \frac{1}{2\sqrt{x(1-x)}} = \frac{1}{2\sqrt{x-x^2}}$

5) $y = \ln(\sin^{-1}x)$

$y = \frac{1}{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}$