

DETERMINANTS. CRAMER'S RULE

$$\text{Ex: } \begin{cases} 2x - 4y = -8 \\ 3x - 2y = 0 \end{cases} \quad \left[\quad \right]$$

$$\Delta = \begin{vmatrix} 2 & -4 \\ 3 & -2 \end{vmatrix} = 2(-2) - (-4) \cdot 3 \\ = -4 + 12 = 8$$

$$\Delta_x = \begin{vmatrix} -8 & -4 \\ 0 & -2 \end{vmatrix} = (-8)(-2) - (-4) \cdot 0 \\ = 16$$

$$\Delta_y = \begin{vmatrix} 2 & -8 \\ 3 & 0 \end{vmatrix} = 2 \cdot 0 - (-8) \cdot 3 = 24$$

CRAMER'S RULE

$$x = \frac{\Delta_x}{\Delta} = \frac{16}{8} = 2$$

$$y = \frac{\Delta_y}{\Delta} = \frac{24}{8} = 3$$

Ex:
$$\begin{cases} x + y + z = 3 \\ 2x - 3y + z = -2 \\ 3x - y - z = 9 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 3 - 2 + 3 - (-9 - 1 - 2)$$

$$= 4 - (-12) = 4 + 12 = 16$$

$-9 \leftarrow \begin{matrix} 1 & 1 & 1 \\ 2 & -3 & 1 \end{matrix} \rightarrow +3$
 $-1 \leftarrow \begin{matrix} 1 & 1 & 1 \\ 2 & -3 & 1 \end{matrix} \rightarrow -2$
 $ \leftarrow \begin{matrix} 1 & 1 & 1 \\ 2 & -3 & 1 \end{matrix} \rightarrow +3$

$$\Delta_x = \begin{vmatrix} 3 & 1 & 1 \\ -2 & -3 & 1 \\ 9 & -1 & -1 \end{vmatrix} = 9 + 2 + 9 - (-27 - 3 + 2)$$

$$= 20 - (-28) = 20 + 28 = 48$$

$-27 \leftarrow \begin{matrix} 3 & 1 & 1 \\ -2 & -3 & 1 \end{matrix} \rightarrow +9$
 $-3 \leftarrow \begin{matrix} 3 & 1 & 1 \\ -2 & -3 & 1 \end{matrix} \rightarrow +2$
 $+2 \leftarrow \begin{matrix} 3 & 1 & 1 \\ -2 & -3 & 1 \end{matrix} \rightarrow +9$

$$x = \frac{\Delta_x}{\Delta} = \frac{48}{16} = 3$$

$$\Delta_y = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -2 & 1 \\ 3 & 9 & -1 \end{vmatrix} = 2 + 18 + 9 - (-6 + 9 - 6)$$

$$= 29 - (-3) = 32$$

$-6 \leftarrow \begin{matrix} 1 & 3 & 1 \\ 2 & -2 & 1 \end{matrix} \rightarrow +2$
 $+9 \leftarrow \begin{matrix} 1 & 3 & 1 \\ 2 & -2 & 1 \end{matrix} \rightarrow +18$
 $-6 \leftarrow \begin{matrix} 1 & 3 & 1 \\ 2 & -2 & 1 \end{matrix} \rightarrow +9$

$$y = \frac{\Delta_y}{\Delta} = \frac{32}{16} = 2$$

$$\Delta z = \begin{vmatrix} 1 & 1 & 3 \\ 2 & -3 & -2 \\ 3 & -1 & 9 \end{vmatrix} = -27 - 6 - 6 - (-27 + 2 + 18)$$

$$= -39 - (-7) = -32$$

$$\begin{array}{ccc} \begin{array}{c} -27 \\ +2 \\ +18 \end{array} & \begin{array}{c} 1 & 1 & 3 \\ 2 & -3 & -2 \\ 3 & -1 & 9 \end{array} & \begin{array}{c} -27 \\ -6 \\ -6 \end{array} \end{array}$$

$$z = \frac{\Delta z}{\Delta} = \frac{-32}{16} = -2$$

Solution: (3, 2, -2)