RATIONAL FUNCTIONS: Domains, Holes and Asymptotes

$$R(x) = \frac{P(x)}{q(x)}$$

Ex: $R(x) = \frac{3x^2-5}{x+4}$ $\quad x+4=0 \Rightarrow x=-4$

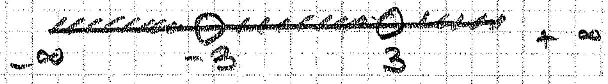
Domain $= \{x \,/\, x \neq -4\} = (-\infty,-4) \cup (-4,+\infty)$

Ex: $R(x) = \frac{3}{x^2-9}$ $\quad x^2-9=(x+3)(x-3)=0 \Rightarrow$

$\Rightarrow x+3=0$ or $x-3=0$

$x=-3$ $\quad x=3$

Domain $= \{x \,/\, x \neq -3,3\} = (-\infty,-3) \cup (-3,3) \cup (3,+\infty)$



Ex: $R(x) = \frac{x^5}{x^2+25}$ $\quad x^2+25=0$ has no real solution

Domain $= \mathbb{R} = (-\infty,+\infty)$

Ex: $R(x) = \frac{x^2-25}{x+5}$ $\quad x+5=0 \Rightarrow x=-5$

Domain $= \{x \,/\, x \neq -5\} = (-\infty,-5) \cup (-5,+\infty)$

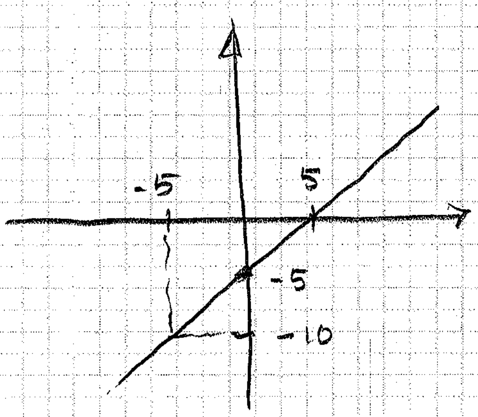
NOTE $\quad \frac{x^2-25}{x+5} = \frac{\cancel{(x+5)}(x-5)}{\cancel{(x+5)}} = x-5$

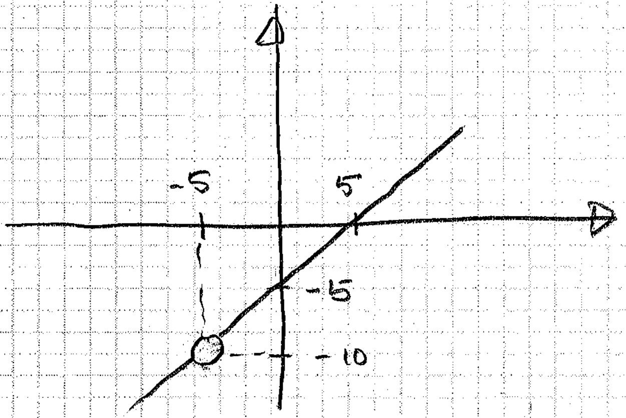
$R(x) = \frac{x^2-25}{x+5}$ and $f(x) = x-5$

are not the same function

the graph of $f(x) = x-5$ is a straight line

the graph of $R(x) = \frac{x^2-25}{x+5} = \frac{(x+5)(x-5)}{x+5}$ is a line with a hole

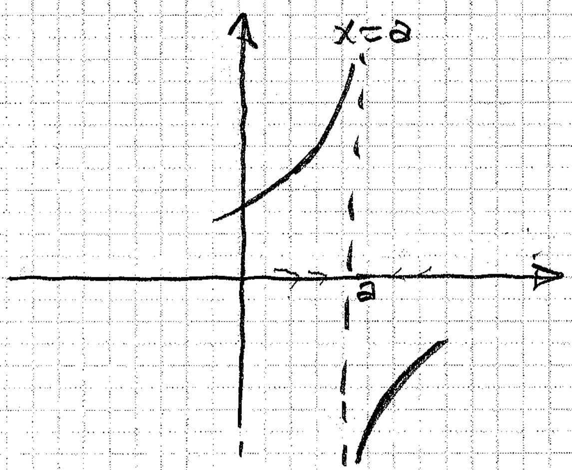


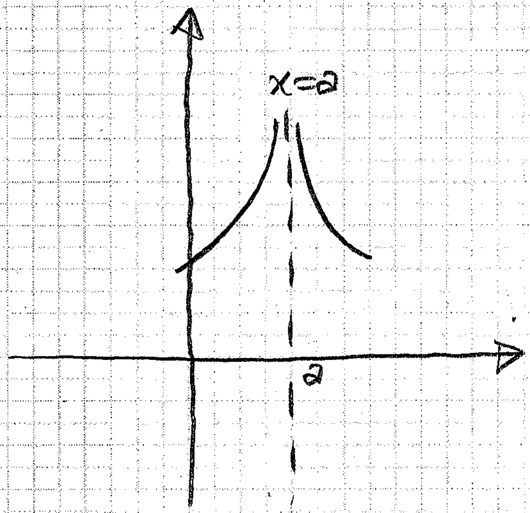


LOWEST TERMS

If $R(x) = \frac{p(x)}{q(x)}$ and $p(x)$ and $q(x)$ have no common factors, then $R(x)$ is said to be in lowest terms

VERTICAL ASYMPTOTES





If when $x \to a$ $y \to +\infty$ or $-\infty$ then $x=a$ is a Vertical Asymptote of the function $R(X)$

If $R(X) = \frac{P(X)}{q(X)}$ is in lowest terms, $R(X)$ has a vertical asymptote wherever $q(X) = 0$

Ex: $R(X) = \frac{3}{x^2-9} = \frac{3}{(x+3)(x-3)}$

$(x+3)(x-3)=0$

$x+3=0$ or $x-3=0$

$x=-3$ $x=3$

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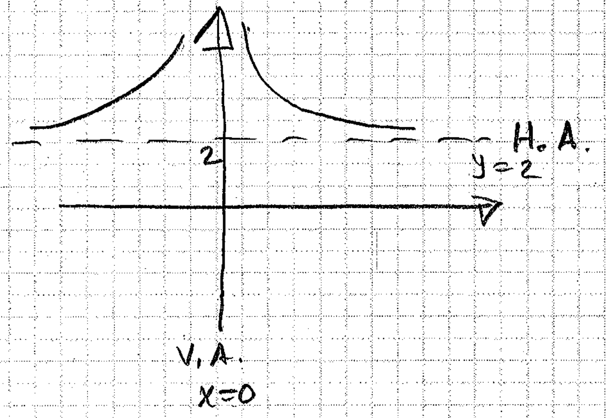
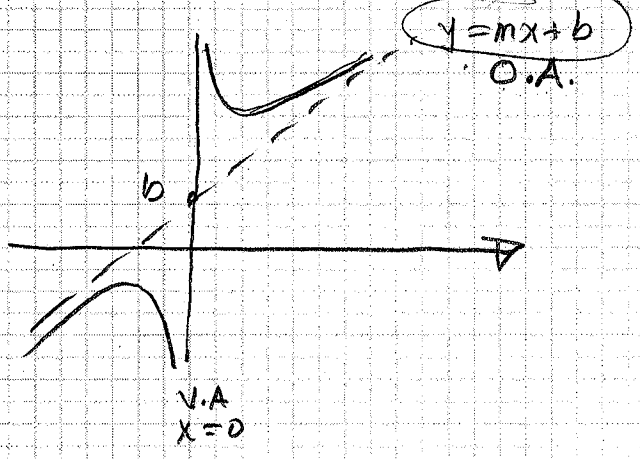
Ex: $R(X) = \frac{x^2-25}{x+5} = \frac{(x+5)(x-5)}{x+5}$

$R(X)$ in lowest terms is: $R(X) = X-5$

NO V.A.

HORIZONTAL AND OBLIQUE ASYMPTOTES





Ex: $$R(x)=\frac{2x^3+3x^2-5x+2}{5x^3+x-7}\begin{matrix}\leftarrow \text{degree } n=3\\ \leftarrow \text{degree } m=3\end{matrix}$$

Since $n=m$, $R(x)$ has a H. A. at $y=\frac{2}{5}$ and <u>no</u> O. A.

Ex: $$R(x)=\frac{(x+2)(x-3)}{(x+1)(x+3)(x+5)}\begin{matrix}\leftarrow \text{degree } n=2\\ \leftarrow \text{degree } m=3\end{matrix}$$

Since $n<m$, $R(x)$ has a H. A. at $y=0$

Ex: $R(x) = \dfrac{2x^4 - 3x^2}{x^3 - 2x^2 + 3}$ ⟵ degree $n = 4$, ⟵ degree $m = 3$

Since $n = m+1$, $R(x)$ has an O.A. and to find it we must do a long division

$$\begin{array}{r}\text{Quotient} \\ \text{Divisor} \,\big|\, \text{Dividend} \\ \text{Remainder}\end{array}$$

$$\begin{array}{r|l} & \boxed{2x + 2} \\ \hline x^3 - 2x^2 + 3 & 2x^4 + 0x^3 - 3x^2 + 0x \\ & -\,\cancel{2x^4} - 2x^3 + 0x^2 + 6x \\ \hline & +2x^3 - 3x^2 - 6x \\ & -\,2x^3 - 4x^2 + 0x + 6 \\ \hline & x^2 - 6x - 6 \end{array}$$

$\dfrac{+2x^4}{+x^3} = +2x$

$\dfrac{+2x^3}{+x^3} = 2$

O.A:

$y = 2x + 2$

$x = 0 \quad y = 2$

$y = 0 \quad 2x + 2 = 0$

$2x = -2$

$x = -1$