EXPECTED VALUE AND STANDARD DEVIATION OF DISCRETE RANDOM VARIABLES

Example

x	$P(x)$
0	1/8
1	1/4
2	1/2
3	1/8

$$\text{Mean} = \mu = 0 * \frac{1}{8} + 1 * \frac{1}{4} + 2 * \frac{1}{2} + 3 * \frac{1}{8}$$

Mean $= \mu =$ Expected Value $= E(X)$

$$E(X) = \sum [x \cdot P(x)]$$

Example:

Mean of a set of numbers

1, 2, 3, 4, 5, 6

$$\bar{x} = \frac{\sum x}{n} =$$

$$= \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

x	$P(x)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

$$E(X) = 1 * \frac{1}{6} + 2 * \frac{1}{6} + 3 * \frac{1}{6} + 4 * \frac{1}{6} + 5 * \frac{1}{6} + 6 * \frac{1}{6}$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$

$$= \frac{1+2+3+4+5+6}{6} = 3.5$$

STANDARD DEVIATION

For a set of numbers 4, 6, 8, 10, 12 (sample)

x	$x-\bar{x}$	$(x-\bar{x})^2$
4	$4-8=-4$	16
6	$6-8=-2$	4
8	$8-8=0$	0
10	$10-8=2$	4
12	$12-8=4$	16

$$S=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

$$S=\sqrt{\frac{\sum x^2-\frac{(\sum x)^2}{n}}{n-1}}$$

shortcut Formula

$$\bar{x}=\frac{4+6+8+10+12}{5}=8$$

For a Discrete Distribution

x	$P(x)$
x_1	$P(x_1)$
x_2	$P(x_2)$
$\vdots$	
x_n	$P(x_n)$

$$\sigma=\sqrt{\sum(x-\mu)^2 \cdot p(x)}$$

$$\sigma=\sqrt{\sum[x^2p(x)]-\mu^2}$$

<u>Example</u>: Calculate the standard deviation of the following distribution

x	$p(x)$	$x\,p(x)$	$x^2p(x)$
0	1/8	0	0
1	1/4	1/4	1/4
2	1/2	1	2
3	1/8	3/8	9/8
		13/8	27/8

$$\sigma=\sqrt{\frac{27}{8}-\left(\frac{13}{8}\right)^2}$$

$\approx .86$

variance $=V(x)=\sigma^2$

$=\frac{47}{64}\approx .73$

$$\mu=E(x)=\sum[x\,p(x)]=13/8$$