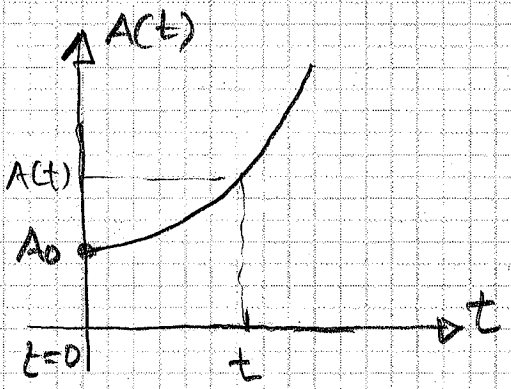
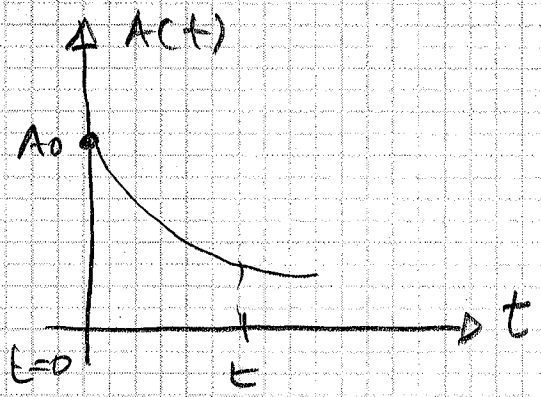
EXPONENTIAL GROWTH AND DECAY



Uninhibited Growth

$A(t) = A_0 \cdot e^{kt}$

k is a positive constant



$A(t) = A_0 \cdot e^{kt}$

k is a negative constant

$P(t) = P_0 e^{kt}$

$N(t) = N_0 e^{kt}$

EX: If a colony of bacteria follows the Law of Uninhibited Growth and the number of bacteria doubles every 5 hours, what is the value of the constant k?

$N(t) = N_0 e^{kt}$

$2N_0 = N_0 e^{5k}$

$2 = e^{5k}$

$\ln 2 = \ln e^{(5k)}$

$\ln 2 = 5k \ln e^{1}$

$\ln 2 = 5k$

$\frac{\ln 2}{5} = k \approx .1386\ldots$

$\ln e = \log_e e$?

$$N(t) = N_0 e^{\frac{\ln 2}{5} t}$$

Ex: If the initial amount of bacteria (in grams) is 50, how many (in grams) will we have in 6 hours?

$$N(t) = N_0 e^{\frac{\ln 2}{5} t}$$

$$N(t) = 50 e^{\frac{\ln 2}{5} t}$$

$$N(6) = 50 e^{\frac{\ln 2}{5} \cdot 6}$$

$$\approx 115 \text{ grams}$$

Ex: How long will it take to triple the number of bacteria?

$$N(t) = N_0 e^{\frac{\ln 2}{5} \cdot t}$$

$$3N_0 = N_0 e^{\frac{\ln 2}{5} \cdot t}$$

$$3 = e^{\frac{\ln 2}{5} \cdot t}$$

$$\ln 3 = \ln e^{\left(\frac{\ln 2}{5} \cdot t\right)}$$

$$\ln 3 = \frac{\ln 2}{5} \cdot t \cdot \ln e^{1}$$

$$\frac{\ln 3}{\frac{\ln 2}{5}} = t$$

$$t \approx 7.92 \text{ days}$$

Ex: The half-life of carbon 14 is 5600 years. If 50 grams is present now, how much will be present in 1000 years?

$$A(t) = A_0 e^{kt}$$

$$\frac{A_0}{2} = A_0 e^{5600k}$$

$$\frac{1}{2} = e^{5600k}$$

$$\ln\left(\frac{1}{2}\right) = \ln e^{(5600k)}$$

$$\ln\left(\frac{1}{2}\right) = 5600k \cdot \ln e^{1}$$

$$\boxed{\frac{\ln(1/2)}{5600}} = k \approx -.0001\ldots$$

$$A(t) = A_0 e^{\frac{\ln(.5)}{5600} t}$$

$$A(1000) = 50 e^{\frac{\ln(.5)}{5600} \times 1000}$$

$$\approx 44 \text{ years}$$