FACTORING

$2 \cdot 3 \cdot 7$

$\nearrow \quad \uparrow \quad \uparrow$

factors

"factoring" means turning additions and subtractions into products.

CASE 1 $x^2 - a^2 = (x+a)(x-a)$

$(x+a)(x-a) = x^2 - ax + ax - a^2 = x^2 - a^2$

Ex: Factor $x^2 - 25 = (x+5)(x-5)$

$\downarrow \quad \downarrow$

$x \quad 5$

Ex: Factor $4x^2 - 36 = (2x+6)(2x-6)$

$\downarrow \quad \downarrow$

$2x \quad 6$

Ex: Factor $64 - 9x^2 = (8+3x)(8-3x)$

$\downarrow \quad \downarrow$

$8 \quad 3x$

CASE 2: Perfect Squares

$x^2 + 2x + 1 = (x+1)^2 = (x+1)(x+1)$

$x \quad 1$

$= x^2 + x + x + 1 = x^2 + 2x + 1$

$x^2 - 6x + 9 = (x-3)^2$

$x \quad 3$

$9x^2 - 6x + 1 = (3x-1)^2$

$3x \quad 1$

$25x^2 + 30x + 9 = (5x+3)^2$

$5x \quad 3$

$2 \cdot 5x \cdot 3 = 30x$

CASE 3 Sums and Differences of Cubes

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$

Ex: Factor

$$x^3 - 125 = (x-5)(x^2 + 5x + 25)$$

$x \quad 5$

Ex: factor

$$8 + 27x^3 = (2+3x)(4 - 6x + 9x^2)$$

$2 \quad 3x$

CASE 4: Factoring a second Degree Polynomial where the leading coefficient is 1

$$x^2 + bx + c$$

EX $x^2 + 5x + 6 = (x+2)(x+3)$

- $= c = 6 = 6,1 \quad -6,-1 \quad (2,3) \quad -2,-3$
- $+ = b = 5 \quad 7 \quad -7 \quad 5 \quad -5$

EX: $-x^2 + 7x - 12 = -(x^2 - 7x + 12)$

- $= c = 12 \quad 12,1 \quad -12,-1 \quad 4,3 \quad (-4,-3) \quad 6,2 \quad -6,-2$
- $+ = b = -7$

$$= -(x-4)(x-3)$$

CASE 5: Factoring by Grouping

EX: $x^3 - 7x^2 - 3x + 21$

$x^2(x-7) - 3(x-7) = (x-7)(x^2-3)$

CASE 6 Factoring a second degree polynomial where the leading coefficient, a, is not 1

EX: $2x^2 + 5x + 3$ $\quad a=2 \quad b=5 \quad c=3$

$ax^2 + bx + c$

$\bullet = a \cdot c = 6 \quad 6,1 \quad -6,-1 \quad (2,3) \quad -2,-3$

$+ = b = 5$

$2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$ (2x·1, 3·1)

$= 2x(x+1) + 3(x+1)$

$= (x+1)(2x+3)$