FOUR BASIC LIMITS

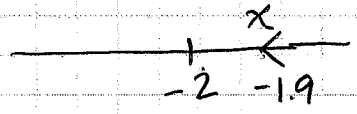
1) $\lim_{x \to 0^+} \frac{1}{x} = +\infty$

2) $\lim_{x \to 0^-} \frac{1}{x} = -\infty$

3) $\lim_{x \to +\infty} \frac{1}{x} = 0$

4) $\lim_{x \to -\infty} \frac{1}{x} = 0$

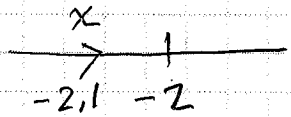
Ex: $\lim_{x \to -2^+} \frac{3}{x+2} = +\infty$ ($3 \to +$, $x+2 \to 0^+$)



Ex: $\lim_{x \to -2^+} \frac{-3}{x+2} = -\infty$ ($-3 \to -$, $x+2 \to 0^+$)

Ex: $\lim_{x \to -2^-} \frac{-3}{x+2} = +\infty$ ($-3 \to -$, $x+2 \to 0^-$)

$\Rightarrow \lim_{x \to -2} \frac{-3}{x+2}$ does not exist d.n.e.



Ex: $\lim_{x \to -2^-} \frac{x+1}{x+2} = +\infty$ ($x+1 \to -1$, $x+2 \to 0^-$)

NOTE

$\frac{12}{4} = 3$ because 3 is the only number that multiplied by 4 gives 12

$\frac{0}{4} = 0$ because zero is the only number that multiplied by 4 gives zero

$\frac{12}{0}$ = undefined because there is no number that multiplied by zero gives 12

$\frac{0}{0} = 0$ or $\frac{0}{0} = 3$ $\frac{0}{0} = 5$

$\frac{0}{0}$ is an indetermination.

$\frac{3}{0}$ undefined $\lim_{x \to 0^+} \frac{1}{x} = +\infty$

Ex: $\lim_{x \to -2} \frac{x^2 + 5x + 6}{x + 2}$ (4, −10, 6 → 0; $x+2 \to 0$)

FACTOR!

you must do SOMETHING to eliminate the indetermination.

$= \lim_{x \to -2} \frac{(x+2)(x+3)}{(x+2)} = \lim_{x \to -2} (x+3) = 1$

EX: $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$ ($\sqrt{x}-3 \to 0$, $x-9 \to 0$)

CONJUGATE!

$(a+b)(a-b) = a^2-b^2$

$(\sqrt{x}-3)(\sqrt{x}+3) = (\sqrt{x})^2 - 3^2 = x-9$

$$\lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)}$$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{3+3} = \frac{1}{6}$$

EX: $\lim_{x\to+\infty} (3x^2 - 5x + 6) = \lim_{x\to+\infty} (3x^2) = +\infty$ ($3x^2 \to +\infty$, $-5x \to -\infty$)

which term "dominates"

EX: $\lim_{x\to+\infty} \dfrac{3x^2-5x+6}{2x^2-9x+5} = \lim_{x\to+\infty} \dfrac{3x^2}{2x^2} =$

$$= \lim_{x\to+\infty} \frac{3}{2} = \frac{3}{2}$$

EX: $\lim_{x\to-\infty} \dfrac{3x^2-5x+6}{2x^3-9x^2+4} = \lim_{x\to-\infty} \dfrac{3x^2}{2x^3}$

$$= \lim_{x\to-\infty} \frac{3}{2x} = \frac{3}{2} \lim_{x\to-\infty} \frac{1}{x} = -\frac{3}{2}\cdot 0 = 0$$

($\frac{1}{x} \to 0$)

Ex: $\lim_{x\to-\infty} \dfrac{3x^2-5x+6}{2x^3-9x^2+4} =$

$$\lim_{x\to-\infty} \frac{\frac{3x^2}{x^3}-\frac{5x}{x^3}+\frac{6}{x^3}}{\frac{2x^3}{x^3}-\frac{9x^2}{x^3}+\frac{4}{x^3}}$$

$$= \lim_{x\to-\infty} \frac{\frac{3}{x}-\frac{5}{x^2}+\frac{6}{x^3}}{2-\frac{9}{x}+\frac{4}{x^3}} = \frac{0}{2} = 0$$

($\frac{3}{x}\to 0$, $\frac{5}{x^2}\to 0$, $\frac{6}{x^3}\to 0$; $\frac{9}{x}\to 0$, $\frac{4}{x^3}\to 0$)