GRAPHING RATIONAL FUNCTIONS

$$R(x) = \frac{x^3 + x^2}{x^3 + 2x^2 - 5x - 6} = \frac{x^2(x+1)}{(x^2+x-6)(x+1)} = \frac{x^2(x+1)}{(x+3)(x-2)(x+1)}$$

1) Factor numerator and denominator

$x^3 + x^2 = x^2(x+1)$

	1	2	-5	-6
-1		-1	-1	6
	1	1	-6	0

$x^2 + x - 6 = (x+3)(x-2)$

$\bullet = -6$

$+ = +1$

2) Find the domain

Domain $= \{x \mid x \neq -3, 2, -1\}$

3) Put $R(x)$ in lowest terms

$$R(x) = \frac{x^2}{(x+3)(x-2)}$$

4) Find Vertical Asymptotes

$\boxed{x = -3}$ $\boxed{x = 2}$

5) At $x = -1$ the graph of $R(x)$ has a hole

$$R(-1) = \frac{(-1)^2}{(-1+3)(-1-2)} = \frac{1}{2(-3)} = -\frac{1}{6} \qquad (-1, -1/6)$$

6) Horizontal and oblique Asymptotes

$$R(x) = \frac{x^2}{(x+3)(x-2)} = \frac{1 \cdot x^2}{1x^2 + x - 6}$$

← degree $n = 2$

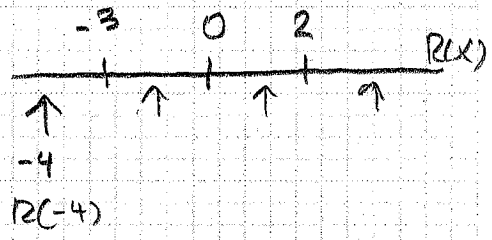
← degree $m = 2$

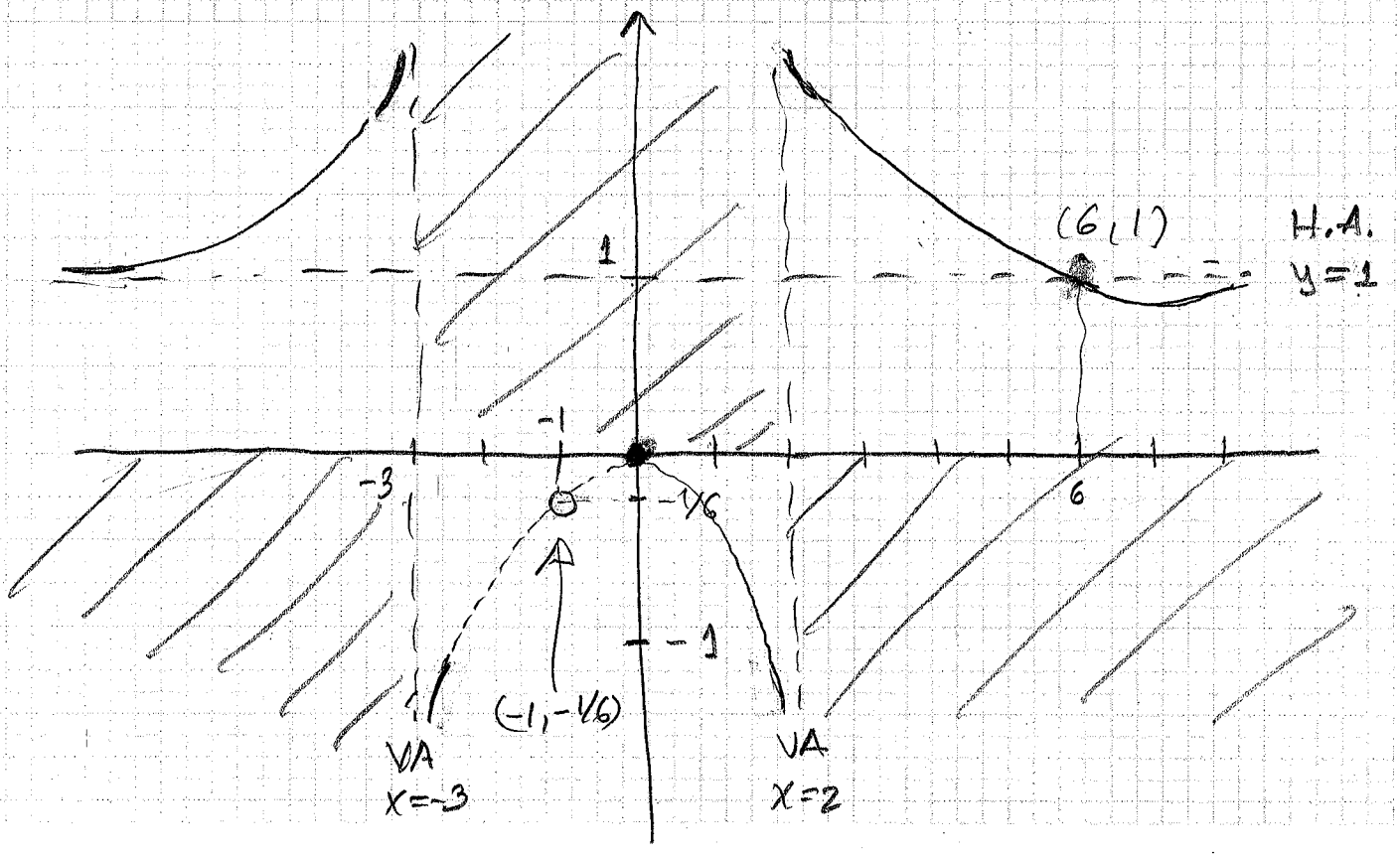
Since $n = m$, $R(x)$ has a H.A. at $y = \frac{1}{1} = 1$

7) Find the signs of $R(x)$; i.e., in what intervals is $R(x)$ above the x-axis and in what intervals is $R(x)$ below the x-axis

	-3		0		2	
x^2	+	+	0	+	+	$x^2 = 0 \Rightarrow x = 0$
$x+3$	−	+	+	+	+	$x+3 = 0 \Rightarrow x = -3$
$x-2$	−	−	−	−	+	$x-2 = 0 \Rightarrow x = 2$
$R(x)$	+	−	−	−	+	

$-\infty$ -3 0 2 $+\infty$





8) Intercepts

y-intercept. set $x=0$ in $y = \frac{x^2}{x^2+x-6}$

$$y = \frac{0}{-6} = 0 \qquad (0,0)$$

x-intercepts. set $y=0$ and find x

$$\frac{x^2}{x^2+x-6} = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0 \qquad (0,0)$$

9) Does the graph intersect the horizontal asymptote? $y=1$

$$y = \left| \frac{x^2}{x^2+x-6} = 1 \right| \Rightarrow x^2 = x^2 + x - 6$$

$$0 = x - 6$$

$$\boxed{6 = x}$$