

HALF ANGLE FORMULAS

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(2\theta) + 2\sin^2\theta = 1$$

$$2\sin^2\theta = 1 - \cos(2\theta)$$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1 - \cos\theta}{2}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$\cos(2\theta) + 1 = 2\cos^2\theta$$

$$\frac{\cos(2\theta) + 1}{2} = \cos^2\theta$$

$$\frac{\cos\theta + 1}{2} = \cos^2\left(\frac{\theta}{2}\right)$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos\theta}{2}}$$

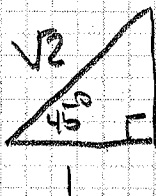
$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \pm \sqrt{\frac{(1 - \cos\theta)/2}{(1 + \cos\theta)/2}} =$$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

ex: Find $\sin 22.5^\circ$

$$22.5 = \frac{45}{2}$$

$$\sin\left(\frac{45}{2}\right) = +\sqrt{\frac{1 - \cos 45^\circ}{2}} = \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}}$$



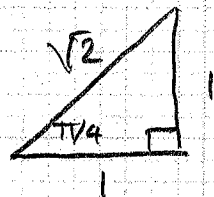
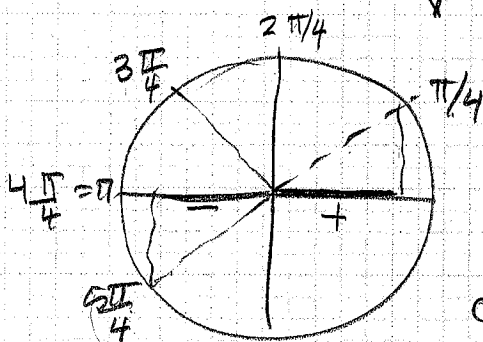
$$= \sqrt{\frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}} \cdot \frac{1}{2}} =$$

$$= \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} = \sqrt{\frac{(\sqrt{2}-1)\sqrt{2}}{2\sqrt{2}\sqrt{2}}} = \sqrt{\frac{2-\sqrt{2}}{4}}$$

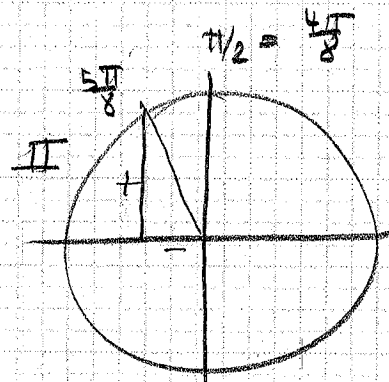
ex: Find $\tan\left(\frac{5\pi}{8}\right)$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$\tan\left(\frac{5\pi}{8}\right) = -\sqrt{\frac{1 - \cos\left(\frac{5\pi}{4}\right)}{1 + \cos\left(\frac{5\pi}{4}\right)}}$$



$$\cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$



$$\tan\left(\frac{5\pi}{4}\right) = -\sqrt{\frac{1 + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}}} = -\sqrt{\frac{\frac{\sqrt{2}+1}{\sqrt{2}}}{\frac{\sqrt{2}-1}{\sqrt{2}}}}$$

$$= -\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$$