

Comparing Two Population Variances

$H_0: \sigma_1^2 = \sigma_2^2$

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$H_a: \sigma_1^2 > \sigma_2^2$

$H_a: \sigma_1^2 < \sigma_2^2$

$H_a: \sigma_1^2 \neq \sigma_2^2$

right-tail test

Left-tail test

two-tail test

two samples

n_1
 s_1^2

n_2
 s_2^2

Conditions

- 1) The samples are random and independent
- 2) Both populations are normally distributed

Test Statistic

$F = \frac{s_1^2}{s_2^2}$

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$F = \frac{\text{Larger Sample Variance}}{\text{Smaller Sample Variance}}$

Rejection Region

$F > F_\alpha$

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$F > F_{\alpha/2}$

$df_{\text{numerator}} = n_1 - 1$

$df_{\text{num}} = n_2 - 1$

$df_{\text{num}} =$

$df_{\text{denominator}} = n_2 - 1$

$df_{\text{den.}} = n_1 - 1$

$df_{\text{den}} =$



Example:

Sample 1

50 48 49 51 52
47 51 51 49 50

$$n_1 = 10$$

$$s_1^2 \approx 1.55^2 \\ = 2.40$$

Sample 2

46 50 48 54 56
45 48 53

$$n_2 = 8$$

$$s_2^2 = 3.95^2 \\ = 15.68$$

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_a: \sigma_1^2 \neq \sigma_2^2$$

two-tailed test

$$\alpha = .05 \Rightarrow \alpha/2 = .025$$

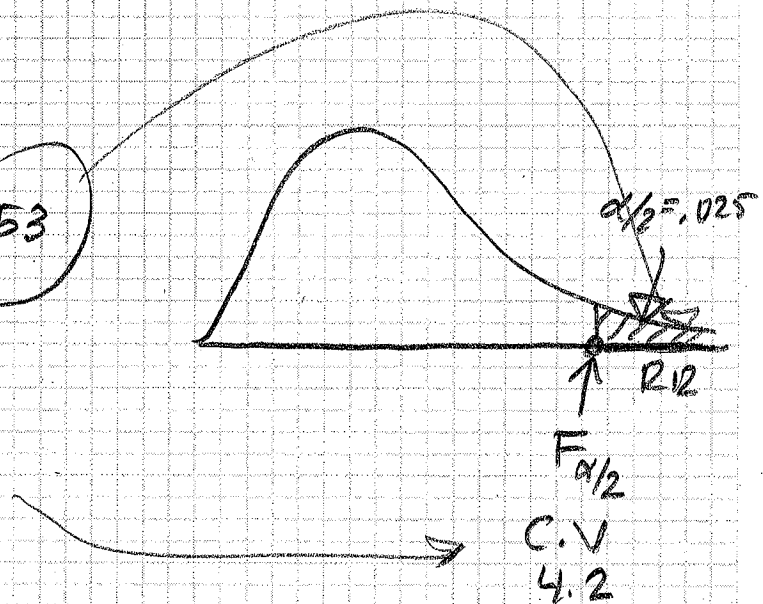
test stat

$$F = \frac{15.68}{2.40} \approx 6.53$$

$$\alpha/2 = .025$$

$$df_{num} = n_2 - 1 = 8 - 1 = 7$$

$$df_{denom} = n_1 - 1 = 10 - 1 = 9$$



$$RR: F > 4.2$$

Decision: Reject H_0

Conclusion: the data provide sufficient evidence to conclude that the two population variances are different