Hypothesis Test about a Population Variance

Left-tail test	Right-tail test	Two-tail test
$H_0: \sigma^2 = \sigma_0^2$	$H_0: \sigma^2 = \sigma_0^2$	$H_0: \sigma^2 = \sigma_0^2$
$H_a: \sigma^2 < \sigma_0^2$	$H_a: \sigma^2 > \sigma_0^2$	$H_a: \sigma^2 \neq \sigma_0^2$

Example: 5.02 5.00 5.05 4.98 4.99
5.06 4.96 5.04 4.97 5.03

Is $\sigma^2 < .01$

Test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \qquad df = n-1$$

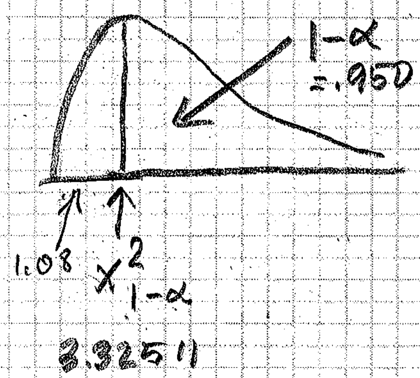
$$s^2 = \frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1} = \frac{251.012 - \frac{50.1^2}{10}}{9} = .0012$$

x	x^2
5.02	25.2004
5.00	25.0
⋮	
5.03	25.3009
Σx	Σx^2
50.1	251.012

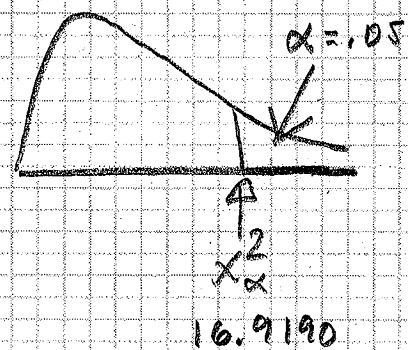
$$\chi^2 = \frac{(10-1)*.0012}{.01} = 1.08$$

Rejection Region $\alpha = .05$ $df = 9$ χ^2 distribution

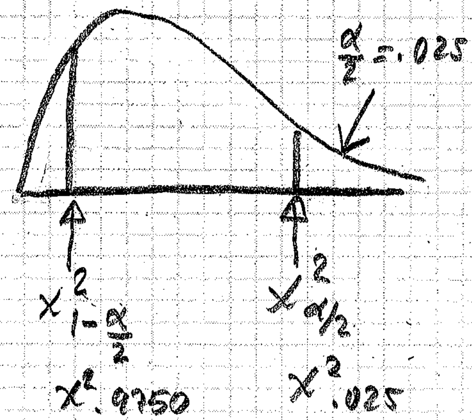
Left-tail Test



Right-tail test



two-tail test



RR: $\chi^2 < 3.32511$

RR: $\chi^2 > 16.9190$

Decision: Reject Ho

Conclusion: the data provide sufficient evidence to conclude, at $\alpha = .05$, that the population variance is less than .01