HYPOTHESIS TEST FOR MEANS WITH PAIRED SAMPLES

Pair	Var x_1	Var x_2	$d = x_1 - x_2$	
1	7	4	$7-4=3$	$= d_1$
2	9	5	$9-5=4$	$= d_2$
3	5	4	$5-4=1$	$\vdots$
4	8	3	$8-3=5$	
5	6	4	$6-4=2$	
6	4	3	$4-3=1$	$= d_6$

$$\bar{d} = \frac{\Sigma d}{n_d} = \frac{3+4+1+5+2+1}{6} \approx 2.67$$

$$S_d = \sqrt{\frac{\Sigma (d-\bar{d})^2}{n_d - 1}} \approx 1.63$$

n_d

$\bar{d}$

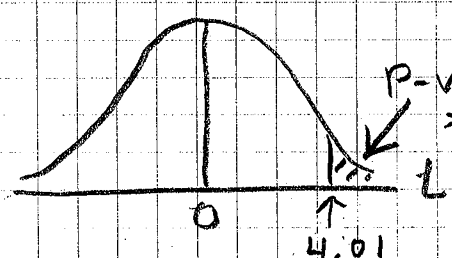
S_d

Small samples

$H_0: \mu_d = 0 \leftarrow D_0$

$H_a: \mu_d > 0$

$$t = \frac{\bar{d} - D_0}{S_d/\sqrt{n}} = \frac{2.67}{1.63/\sqrt{6}} = 4.01$$



$df = n_d - 1 = 6 - 1 = 5$

If we are using $\alpha = .01$ do we reject H_0?

Is p-val $< \alpha$? Yes. We reject H_0

The data provide sufficient evidence to conclude that the mean of the differences is greater than zero.

ONE SAMPLE HYPOTHESIS TEST FOR THE MEAN

Large Samples $H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ Test Statistic: Z = $\dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$	σ is known $H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ Test Statistic: $Z = \dfrac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$
Small Samples Test Statistic: t = $\dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$ With df = n - 1	σ is unknown Test Statistic: t = $\dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$ With df = n - 1

HYPOTHESIS TESTS FOR PAIRED SAMPLES

TWO SAMPLE HYPOTHESIS TEST FOR THE MEAN OF THE DIFFERENCE BETWEEN TWO POPULATION MEANS

Large Samples $H_0 : \mu_d = D_0$ $H_a : \mu_d > D_0$ Test Statistic: Z = $\dfrac{\overline{d} - D_0}{s_d/\sqrt{n}}$	If σ_d is known $H_0 : \mu_d = D_0$ $H_a : \mu_d > D_0$ Test Statistic: Z = $\dfrac{\overline{d} - D_0}{\sigma_d/\sqrt{n}}$
if samples are small Test Statistic: t = $\dfrac{\overline{d} - D_0}{s_d/\sqrt{n_d}}$ where df = $n_d - 1$ The population of differences must be approximately normally distributed.	If σ_d is unknown Test Statistic: t = $\dfrac{\overline{d} - D_0}{s_d/\sqrt{n_d}}$ where df = $n_d - 1$ For most applications a sample size of n $\geq$ 30 is adequate. If the populations distribution is approximately normal, smaller samples may be used. If the distribution is highly skewed, sizes of n $\geq$ 50 are recommended.