HYPOTHESIS TESTS FOR INDEPENDENT SAMPLES

Example:

$n_1 = 40$ $\qquad$ $n_2 = 45$

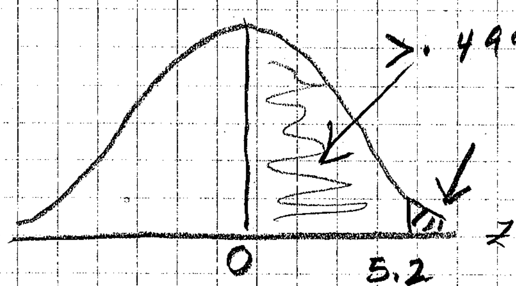
$\bar{x}_1 = 17$ $\qquad$ $\bar{x}_2 = 19$

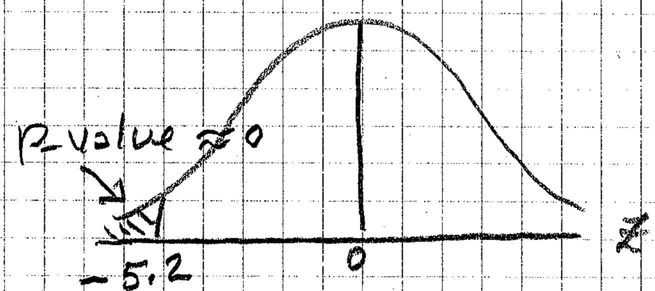
$s_1 = 1.5$ $\qquad$ $s_2 = 2.0$

$H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 < 0$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{17-19}{\sqrt{\frac{1.5^2}{40} + \frac{2.0^2}{45}}} \approx -5.2$$





$.5 - .49997 = .00003$ $\qquad$ using $\alpha = .01$

p-value $< .00003$ $\qquad$ Reject H_0

The samples provide sufficient evidence to conclude, at $\alpha = .01$, that μ_2 is greater than μ_1

Example 2

$n_1 = 15$ $\qquad$ $n_2 = 12$

$\bar{x}_1 = 17$ $\qquad$ $\bar{x}_2 = 19$

$s_1 = 1.5$ $\qquad$ $s_2 = 2.0$

$H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 < 0$

Test statistic $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{14 \times 1.5^2 + 11 \times 2^2}{15 + 12 - 2}$$

$= 3.02$

$$t = \frac{17 - 19}{\sqrt{3.02\left(\frac{1}{15} + \frac{1}{12}\right)}} = -2.97$$

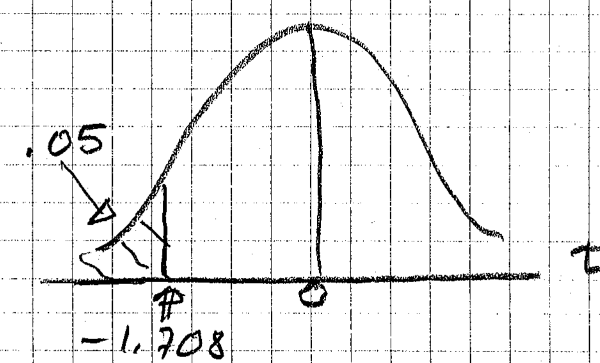
Use $\alpha = .05$

$df = n_1 + n_2 - 2$

$= 15 + 12 - 2 = 25$

RR: $t < -1.708$

Reject H_0.



ONE SAMPLE HYPOTHESIS TEST ABOUT THE MEAN

Large Samples $H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ Test Statistic: Z = $\dfrac{\overline{x} - \mu_0}{s / \sqrt{n}}$	σ is known $H_0 : \mu = \mu_0$ $H_a : \mu > \mu_0$ Test Statistic: $Z = \dfrac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$
Small Samples Test Statistic: t = $\dfrac{\overline{x} - \mu_0}{s / \sqrt{n}}$ With df = n - 1	σ is unknown Test Statistic: t = $\dfrac{\overline{x} - \mu_0}{s / \sqrt{n}}$ With df = n - 1

HYPOTHESIS TEST FOR INDEPENDENT SAMPLES

HIPOTHESIS TEST FOR THE DIFFERENCE BETWEEN TWO POPULATION MEANS

If both samples are large $H_0 : \mu_1 - \mu_2 = D_0$ $H_a : \mu_1 - \mu_2 > D_0$ Test Statistic: Z = $\dfrac{\overline{x}_1 - \overline{x}_2 - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$	If σ_1 and σ_2 are known $H_0 : \mu_1 - \mu_2 = D_0$ $H_a : \mu_1 - \mu_2 > D_0$ Test Statistics: Z = $\dfrac{\overline{x}_1 - \overline{x}_2 - D_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$
if samples are small Test Statistic: t = $\dfrac{\overline{x}_1 - \overline{x}_2 - D_0}{\sqrt{S_p^2(\dfrac{1}{n_1} + \dfrac{1}{n_2})}}$ where: $S_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ And df = $n_1 + n_2 - 2$ Both Samples must be approximately normally distributed with equal population variances and samples must be independent	If σ_1 and σ_2 are unknown Test Statistic: t = $\dfrac{\overline{x}_1 - \overline{x}_2 - D_0}{\sqrt{S_p^2(\dfrac{1}{n_1} + \dfrac{1}{n_2})}}$ where df = $\dfrac{\left(\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{S_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{S_2^2}{n_2}\right)^2}$ not assuming the equality of the variances. For most applications a sample size of $n \geq 30$ is adequate. If the populations distribution is approximately normal, smaller samples may be used. If the distribution is highly skewed, sizes of $n \geq 50$ are recommended.