

IMPLICIT DIFFERENTIATION

$$x^2 + y = 4 \quad \text{Find } \frac{dy}{dx}$$

$$y = 4 - x^2$$

$$\frac{dy}{dx} = -2x$$

$$x^2 y + x y^2 = 2 \quad \text{Find } \frac{dy}{dx}$$

$$\frac{d}{dx} (x^2 y + x y^2) = \frac{d}{dx} (2)$$

product rule

$$\frac{d}{dx} (x^2) \cdot y + x^2 \frac{d}{dx} y + \frac{d}{dx} x \cdot y^2 + x \frac{d}{dx} y^2 = 0$$

NOTES: 1) $\frac{dx}{dx} = 1$

but $\frac{dy}{dx}$ is not necessarily 1. $\frac{dy}{dx}$ is precisely what we are trying to find

2) $\frac{dx^2}{dx} = 2x$ but $\frac{dy^2}{dx} = 2y \cdot \frac{dy}{dx}$
chain rule

$$2xy + x^2 \frac{dy}{dx} + y^2 + x \cdot 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (x^2 + 2xy) = -2xy - y^2$$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy} = -\frac{2xy + y^2}{x^2 + 2xy}$$