Improper Integrals

$$\int_a^b f(x)\,dx$$

$[a,b]$ finite

$f(x)$ has no infinite discontinuities (NO V.A.)

$$\int_1^{+\infty} \frac{dx}{x^2} \qquad \int_{-3}^{3} \frac{dx}{x^2} \qquad \int_{-\infty}^{\infty} \frac{dx}{x^2-9}$$

x^{-2}

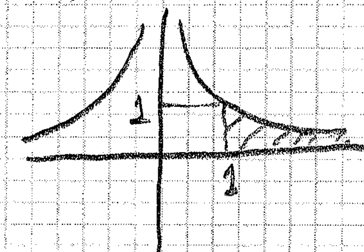
Ex: $$\int_1^{+\infty} \frac{dx}{x^2} = \lim_{t\to+\infty} \int_1^t \frac{dx}{x^2} = \lim_{t\to+\infty} \left.\frac{x^{-1}}{-1}\right|_1^t$$

$$= \lim_{t\to+\infty} \left[-\frac{1}{x}\right]_1^t = -\lim_{t\to+\infty} \left[\frac{1}{x}\right]_1^t = -\lim_{t\to+\infty} \left[\cancel{\frac{1}{t}}^{\,0} - 1\right]$$

$$= -(-1) = 1$$

converges

$$f(x) = \frac{1}{x^2}$$



Ex: $$\int_{-3}^{3} \frac{dx}{x^2} = \cancel{\int_{-3}^{0} \frac{dx}{x^2} + \int_0^3 \frac{dx}{x^2}}$$

$$= \lim_{t\to 0^-} \int_{-3}^{t} \frac{dx}{x^2} + \lim_{t\to 0^+} \int_t^3 \frac{dx}{x^2}$$

$$= \lim_{t\to 0^-}\left[-\frac{1}{x}\right]_{-3}^{t} + \lim_{t\to 0^+}\left[-\frac{1}{x}\right]_{t}^{3}$$

$$= -\lim_{t\to 0^-}\left[\frac{1}{x}\right]_{-3}^{t} - \lim_{t\to 0^+}\left[\frac{1}{x}\right]_{t}^{3}$$

$$= -\lim_{t\to 0^-}\left[\overset{-\infty}{\frac{1}{t}} - \frac{1}{(-3)}\right] - \lim_{t\to 0^+}\left[\frac{1}{3} - \overset{-\infty}{\frac{1}{t}}\right] = +\infty$$

Diverges

$$\cancel{= +\infty - \frac{1}{3} - \left(\frac{1}{3} - \infty\right)}$$

$$\cancel{+\infty - \frac{1}{3} - \frac{1}{3} + \infty} \longrightarrow +\infty$$