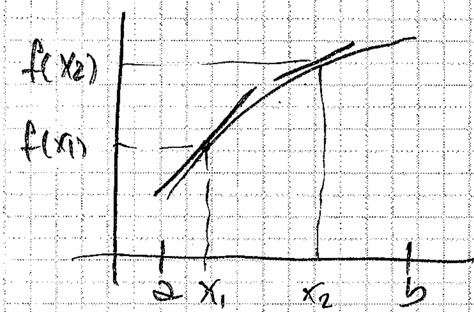
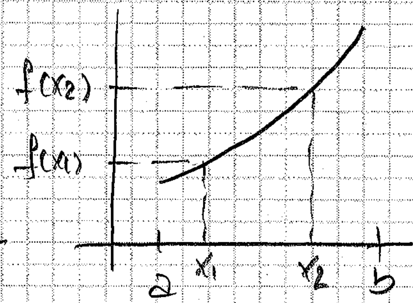
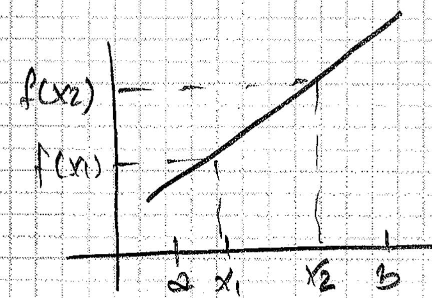
FUNCTIONS: INCREASE, DECREASE

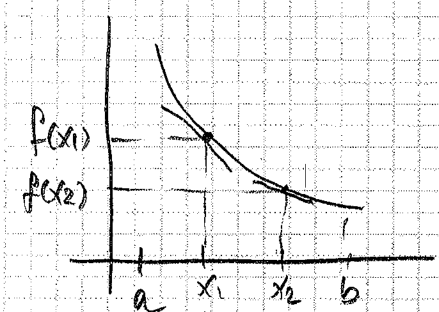


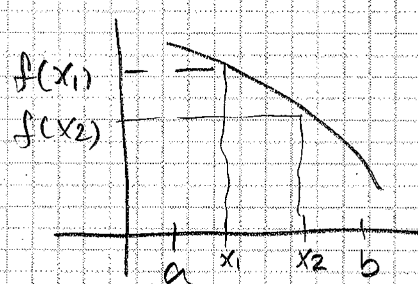


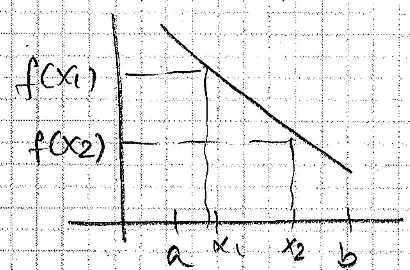


$x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$ f is increasing

$f'(x) > 0$ for all x on (a,b)

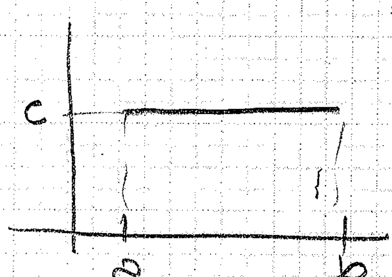






$x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$ f is decreasing

$f'(x) < 0$ for all x on (a,b)



$f(x) = c$ for all x on (a,b)

f is constant

Example: Find the intervals on which f is increasing and the intervals where f is decreasing

$$f(x) = \frac{x^3}{3} - 5\frac{x^2}{2} + 6x \qquad \text{Domain} = (-\infty, +\infty)$$

$$f'(x) = \frac{\cancel{3}x^2}{\cancel{3}} - 5 \cdot \frac{\cancel{2}x}{\cancel{2}} + 6 = x^2 - 5x + 6$$

$$= (x-3)(x-2)$$

$c = 6$ | $-3, -2$ |
$b = -5$

$-\infty$	2		3	∞	
$-$		$-$		$+$	$x-3=0 \Rightarrow x=3$
$-$		$+$		$+$	$x-2=0 \Rightarrow x=2$
$+$		$-$		$+$	$(x-3)(x-2) = f'(x)$

f is increasing on $(-\infty, 2) \cup (3, +\infty)$

f is decreasing on $(2, 3)$

$(x-3)(x-2) = 0$

$x = 3 \quad x = 2$

	2		3		
$+$		$-$		$+$	$f'(x)$

$f'(3) = 3^2 - 5 \cdot 3 + 6 > 0$

$f'(2.5) = 2.5^2 - 5(2.5) + 6 < 0$

$f'(1) = 1^2 - 5 \cdot 1 + 6 > 0$

Example: $f(x) = x^2 \cdot \ln x$ Domain $= (0, +\infty)$

$$f'(x) = 2x \cdot \ln x + x^2 \cdot \frac{1}{x} = 2x\ln x + x =$$

$$= x(2\ln x + 1) = 0 \Rightarrow$$

$$\Rightarrow \cancel{x = 0} \quad \text{or} \quad 2\ln x + 1 = 0$$

$$2\ln x = -1$$

$$\ln x = -\frac{1}{2} \Rightarrow x = e^{-1/2} = \frac{1}{\sqrt{e}} \approx .606$$

0 — $-$ — $1/\sqrt{e}$ — $+$ — $f'(x)$

$f'(1) = 2(1)\ln(1) + 1 = 1$

$f'(.5) = 2(.5)\ln(.5) + .5 < 0$

f is decreasing on $(0, 1/\sqrt{e})$

f is increasing on $(1/\sqrt{e}, +\infty)$

0		$1/\sqrt{e}$	
	$+$	$+$	x
	$-$	$+$	$2\ln x + 1 = 0 \Rightarrow x = 1/\sqrt{e} \approx 0.5$
	$-$	$+$	$f'(x)$

$e \approx 2.71$ $\sqrt{2.71} \approx 1.5$