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INDETERMINATE FORMS

$\frac{0}{0}$ $\frac{\infty}{\infty}$ $0\cdot\infty$ $\infty-\infty$ 1^{∞} ∞^{0} 0^{0}

A limit of the form $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ where $\lim\limits_{x\to a} f(x)=0$

and $\lim\limits_{x\to a} g(x)$ is called an indeterminate form of type $\frac{0}{0}$

why? Example $\lim\limits_{x\to 0}\frac{\cancel{2x}\to 0}{\cancel{x}\to 0} = \lim\limits_{x\to 0}(2)=2$

$\lim\limits_{x\to 0}\frac{\cancel{3x}\to 0}{\cancel{x}\to 0} = 3$

A limit of the form $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ where $\lim\limits_{x\to a} f(x)=\pm\infty$

and $\lim\limits_{x\to a} g(x)=\pm\infty$ is called an indeterminate

form of type $\frac{\infty}{\infty}$ why?

$\lim\limits_{x\to 0^+}\frac{\cancel{\frac{2}{x}}\to+\infty}{\cancel{\frac{1}{x}}\to+\infty} = \lim\limits_{x\to 0^+} 2 = 2$

A limit of the form $\lim\limits_{x\to a}[f(x)\cdot g(x)]$ where

$\lim\limits_{x\to a} f(x)=0$ and $\lim\limits_{x\to a} g(x)=\pm\infty$

is called an indeterminate form of type $0\cdot\infty$

$\lim\limits_{x\to 0^+}\left[\underbrace{(2\cancel{x})}_{0}\underbrace{\left(\frac{1}{\cancel{x}}\right)}_{+\infty}\right] = \lim\limits_{x\to 0^+} 2 = 2$

A limit of the form $\lim_{x \to a} [f(x) - g(x)]$ where

$\lim_{x \to a} f(x) = +\infty$ and $\lim_{x \to a} g(x) = +\infty$ is called an

indeterminate form of type $\infty - \infty$

<u>Notice</u> that $(+\infty) - (-\infty)$ is <u>not</u> an I.F.

or $(-\infty) - (\infty)$ " "

only if we have conflicting influences

$$\lim_{x \to 0^+} \Big[\big(\underbrace{2 + \cancel{\tfrac{1}{x}}}_{+\infty}\big) - \underbrace{\tfrac{1}{x}}_{+\infty} \Big] = \lim_{x \to 0^+} (2) = 2$$

A limit of the form $\lim_{x \to a} [f(x)]^{g(x)}$

where $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \pm\infty$

is called an indeterminate form of type 1^{∞}

$$\lim_{x \to 0^+} \big(\underbrace{1 + x}_{1}\big)^{\frac{\ln 2}{x}} = \lim_{x \to 0^+} \left[(1+x)^{\frac{1}{x}} \right]^{\ln 2} = e^{\ln 2} = 2$$

A limit of the form $\lim_{x \to a} [f(x)]^{g(x)}$ where

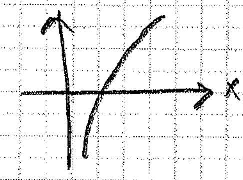
$\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$ is called

an indeterminate form of type 0^0

$$\lim_{x \to 0} [0^x] = \lim_{x \to 0} 0 = 0$$

$$\lim_{x \to 0} [x^0] = \lim_{x \to 0} 1 = 1$$

$$\lim_{x\to 0} \left(\underset{\to 0}{x}\right)^{\overset{\to 0}{\frac{\ln 2}{\ln x}}} = \lim_{x\to 0}\left[x^{\frac{1}{\ln x}}\right]^{\ln 2} = \lim_{x\to 0} e^{\ln 2} = \lim_{x\to 0} 2 = 2$$



$$y = x^{\frac{1}{\ln x}} \Rightarrow \ln y = \ln x^{\frac{1}{\ln x}} \Rightarrow$$

$$\Rightarrow \ln y = \frac{1}{\ln x}\cdot \ln x \Rightarrow \ln y = 1 \Rightarrow y = e$$

A limit of the form $\lim_{x\to a}[f(x)]^{g(x)}$ where

$\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = 0$ is called an

indeterminate form of type ∞^0 why?

$$L = \lim_{x\to +\infty} \left(\underset{\to\infty}{x}\right)^{\overset{\to 0}{\frac{\ln 2}{\ln x}}} = \lim_{x\to +\infty} 2 = 2$$