INFINITE SEQUENCES. PART III

Ex: $\left\{ \dfrac{5^n}{n!} \right\}_{n=1}^{\infty}$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{5^{n+1}}{(n+1)!}}{\dfrac{5^n}{n!}} = \frac{5^{n+1}}{(n+1)!}\,\frac{n!}{5^n} = \frac{\cancel{5^n}\cdot 5\cdot \cancel{n!}}{(n+1)\cancel{n!}\,\cancel{5^n}} = \frac{5}{n+1} < 1$$

for all $n \geq 5$

the sequence is strictly decreasing for $n \geq 5$.

The sequence is "eventually" strictly decreasing

If discarding some terms from the beginning of a sequence, a property holds, we say that the sequence "eventually" has that property.

If a number M is greater than or equal than a_n for all n, M is an upper bound

If M is less than or equal than a_n for all n, M is a lower bound

Theorem: If a sequence $\{a_n\}$ is eventually increasing then, either

1) $\{a_n\}$ has an upper bound M and $\lim_{n\to\infty} a_n = L \leq M$

or 2) there is no upper bound and $\lim_{n\to\infty} a_n = +\infty$

Theorem: If a sequence $\{a_n\}$ is eventually decreasing then, either

1) $\{a_n\}$ has a lower bound M and $\lim_{n\to\infty} a_n = L \geq M$

or

2) There is no lower bound and $\lim_{n\to\infty} a_n = -\infty$

Example: Determine if the sequence $\left\{\frac{5^n}{n!}\right\}_{n=1}^{\infty}$ converges.

the sequence is eventually decreasing

$M=0$ is a lower bound

$$\frac{a_{n+1}}{a_n} = \frac{5}{n+1} \implies a_{n+1} = \frac{5}{n+1} \cdot a_n$$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\left(\frac{5}{n+1}\cdot a_n\right) = \overbrace{\lim_{n\to\infty}\frac{5}{n+1}}^{0} \cdot \lim_{n\to\infty} a_n = 0$$

$$\boxed{\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n = L}$$

$\lim_{n\to\infty} a_n = 0$ the sequence converges to zero.

NEXT: Infinite series