INFINITE SERIES

An expression of the form:

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

is called an infinite series and is denoted by $\sum_{n=1}^{\infty} a_n$

$$\left.\begin{aligned} s_1 &= a_1 \\ s_2 &= a_1 + a_2 \\ s_3 &= a_1 + a_2 + a_3 \\ &\vdots \\ s_n &= a_1 + a_2 + a_3 + \cdots + a_n \end{aligned}\right\} \text{partial sums}$$

$\{s_1, s_2, s_3, \ldots, s_n, \ldots\} = \{s_n\}$ = sequence of Partial Sums

s_n ← the n^{th} partial sum

If the sequence $\{s_n\}$ is convergent and $\lim_{n\to\infty} s_n = S$ then we say that the series $\sum_{n=1}^{\infty} a_n$ converges and we write $\sum_{n=1}^{\infty} a_n = S$

GEOMETRIC SERIES

$$\sum_{k=0}^{\infty} a \cdot r^k = a + ar + ar^2 + \cdots + ar^k + \cdots \quad (a \neq 0)$$

r = common ratio

$$s_n = \frac{a - a \cdot r^{n+1}}{1-r} = \frac{a}{1-r}\left(1 - r^{n+1}\right)$$

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty}\left[\frac{a}{1-r}\left(1-r^{n+1}\right)\right]$$

if $|r| < 1$ then $r^{n+1} \to 0$ $\quad \lim_{n\to\infty} s_n = \frac{a}{1-r}$

the series converges

if $|r| \geq 1$ the series diverges (why?)

If $r=1$ the series becomes

$$a+a+a+\cdots$$

$$S_n = a + a\cdot r + a\cdot r^2 + \cdots + a\cdot r^n$$
$$= a + a + a + \cdots + a \quad n+1 \text{ times}$$
$$= (n+1)a \qquad \lim_{n\to\infty} S_n = \pm\infty$$ depending on the sign of a

the series diverges

If $r=-1$ the series becomes

$$a - a + a - a + \cdots$$

the sequence of partial sums

$S_1 = a$

$S_2 = a - a = 0$

$S_3 = a - a + a = a$

$\vdots$

$a, 0, a, 0, \ldots$ diverges

If $|r| > 1$ then either $r > 1$ or $r < -1$

If $r > 1$ then $\lim_{n\to\infty} r^{n+1} = +\infty$ and $\lim_{n\to\infty} S_n = +\infty$

series diverges

If $r < -1$ r^{n+1} oscillates between positive and negative values that grow in magnitude

series diverges.