INFINITE SERIES 2

TELESCOPING SERIES

EX: $$\sum_{k=1}^{\infty} \frac{3}{k(k+2)} = \frac{3}{1(1+2)} + \frac{3}{2(2+2)} + \frac{3}{3(3+2)} + \cdots$$

$$S_n = \sum_{k=1}^{\infty} \frac{3}{k(k+2)} \qquad \lim_{n\to\infty} S_n = ?$$

$$\frac{3}{k(k+2)} = \frac{A}{k} + \frac{B}{k+2} = \frac{A(k+2) + Bk}{k(k+2)} \Rightarrow$$

$$\Rightarrow \quad 3 = A(k+2) + Bk$$

$k=0 \qquad 3 = 2A \Rightarrow A = 3/2$

$k=-2 \qquad 3 = -2B \Rightarrow B = -3/2$

$$S_n = \sum_{k=1}^{n} \left(\frac{3}{2k} - \frac{3}{2(k+2)} \right)$$

$$= \frac{3}{2} - \cancel{\frac{3}{6}} + \frac{3}{4} - \cancel{\frac{3}{8}} + \cancel{\frac{3}{6}} - \cancel{\frac{3}{10}} + \cancel{\frac{3}{8}} - \cancel{\frac{3}{12}} + \cancel{\frac{3}{10}} - \cancel{\frac{3}{14}} + \cancel{\frac{3}{12}} - \frac{3}{16} + \cancel{\frac{3}{14}} - \frac{3}{18} + \cdots$$

$$\cdots + \cdots$$

$$\frac{3}{2(n-4)} - \cancel{\frac{3}{2(n-2)}} + \frac{3}{2(n-3)} - \cancel{\frac{3}{2(n-1)}} + \cancel{\frac{3}{2(n-2)}} - \cancel{\frac{3}{2n}} + \cancel{\frac{3}{2(n-1)}} - \frac{3}{2(n+1)} + \cancel{\frac{3}{2n}} - \frac{3}{2(n+2)}$$

$$S_n = \frac{3}{2} + \frac{3}{4} - \cancel{\frac{3}{2(n+1)}}^{\to 0} - \cancel{\frac{3}{2(n+2)}}^{\to 0}$$

$$\lim_{n\to\infty} S_n = \frac{3}{2} + \frac{3}{4} = \frac{9}{4}$$ Series converges to $\frac{9}{4}$

A telescoping series is any series where almost all terms cancel

WARNING: Not all telescoping series converge.

For example, the series $\sum_{k=1}^{\infty} [k-(k+1)]$ diverges

$$S_n = \sum_{k=1}^{n} [k-(k+1)] = (1-\cancel{2}) + (\cancel{2}-3) + \cdots + (\cancel{n}-(n+1))$$

$$= 1-(n+1) = -n \Rightarrow \lim_{n\to\infty} S_n = -\infty$$ the series diverges

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Geometric ✓

telescoping ✓

NEXT: Harmonic

p-series