INFINITE SERIES 3. HARMONIC

Geometric ✓ Harmonic Series

Telescopings ✓

THE HARMONIC SERIES

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \quad \text{diverges}$$

$$S_2 = 1 + \frac{1}{2} > \frac{1}{2} + \frac{1}{2} = \frac{2}{2}$$

$$S_{2^2} = S_4 = S_2 + \frac{1}{3} + \frac{1}{4} > S_2 + \frac{1}{4} + \frac{1}{4} = S_2 + \frac{1}{2} > \frac{3}{2}$$

$$S_{2^3} = S_8 = S_4 + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} > S_4 + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = S_4 + \frac{1}{2} > \frac{4}{2}$$

$$S_{2^4} = S_{16} = S_8 + \frac{1}{9} + \frac{1}{10} + \cdots \frac{1}{16} > S_8 + \underbrace{\frac{1}{16} + \frac{1}{16} + \cdots + \frac{1}{16}}_{8 \text{ times}} = S_8 + \frac{1}{2} > \frac{5}{2}$$

$\vdots$

$$S_{2^n} > \frac{n+1}{2} \qquad \text{the Harmonic Series diverges}$$

Ex: Determine if the series converges.

$$\sum_{k=3}^{\infty} \frac{1}{k-2} = \sum_{k=1}^{\infty} \frac{1}{k} \qquad \text{Harmonic} \qquad \text{Diverges}$$