INTEGRALS INVOLVING ax^2+bx+c

$$I = \int \frac{x}{x^2-2x+5}\,dx = \int \frac{x}{(x-1)^2+4}\,dx$$

$x^2 - 2x + 1 + 5 - 1$ $\qquad$ $u = x-1 \Rightarrow x = u+1$

half -1 squared $\qquad$ $du = dx$

$(x-1)^2 + 4$ $\qquad$ $I = \int \frac{u+1}{u^2+4}\,du$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \qquad I = \underbrace{\int \frac{u}{u^2+4}\,du}_{I_1} + \underbrace{\int \frac{du}{u^2+4}}_{I_2}$$

$$I_1 = \int \frac{u}{u^2+4}\,du = \frac{1}{2}\int \frac{2u}{u^2+4}\,du = \frac{1}{2}\int \frac{dw}{w} =$$

$w = u^2+4$

$dw = 2u\,du$

$$= \frac{1}{2}\ln|w| + C_1 = \frac{1}{2}\ln(u^2+4) + C_1$$

$$= \frac{1}{2}\ln\left[(x-1)^2+4\right] + C_1$$

$$I_2 = \int \frac{du}{u^2+4} = \int \frac{1/4}{\left(\frac{u}{2}\right)^2+1}\,du = \frac{1}{4}\,\frac{1}{1/2}\int \frac{1}{\left(\frac{u}{2}\right)^2+1}\,\frac{1}{2}\,du$$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$ $\qquad$ $v = u/2$, $dv = 1/2\,du$

$$= \frac{1}{2}\int \frac{1}{v^2+1}\,dv = \frac{1}{2}\tan^{-1}v + C_2 = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right) + C_2$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right) + C_2 \qquad I = I_1 + I_2$$