Integration by Parts.

$$\int u\,dv = uv - \int v\,du$$

Ex: $I = \int \underbrace{(\ln x)}_{\uparrow}\ \underbrace{x^2\,dx}$

$I = \int \underset{\uparrow u}{x^2}\ \underbrace{(\ln x)\,dx}_{\uparrow dv}$

$u = \ln x$ $\qquad dv = x^2\,dx$

$v = \int x^2\,dx = \frac{x^3}{3}$

$\frac{du}{dx} = \frac{1}{x}$

$\int \frac{1}{x}\,dx = \ln|x| + c$

$\int \ln x\,dx = \ldots ?$

$du = \frac{1}{x}\,dx$

$I = (\ln x)\frac{x^3}{3} - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx$

$= \boxed{\frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx}$

$= \frac{1}{3}x^3\ln x - \frac{1}{3}\frac{x^3}{3} + C$

$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$ done!

Ex: $I = \int \underbrace{\ln x}_{u} \underbrace{dx}_{dv} = uv - \int v\,du$

$u = \ln x \qquad dv = dx$

$\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x}dx \qquad v = x$

$I = (\ln x)\,x - \int x \frac{1}{x}dx$

$= x\ln x - \int dx = \boxed{x\ln x - x + C}$

Ex: $I = \int x e^{3x} dx$

$u = x \qquad dv = e^{3x}dx$

$du = dx \qquad v = \int e^{3x}dx = \frac{e^{3x}}{3}$

$I = uv - \int v\,du$

$= x\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}dx$

$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx$

$= \frac{1}{3}xe^{3x} - \frac{1}{3}\frac{e^{3x}}{3} + C = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$

$I = \int e^{3x}\,x\,dx$

OR $u = e^{3x} \qquad dv = x\,dx$

$du = 3e^{3x}dx \qquad v = \frac{x^2}{2}$

$I = uv - \int v\,du$

$= e^{3x}\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\cdot 3e^{3x}dx$