

INTEGRATION BY SUBSTITUTION

$$\text{ex: } I = \int \sqrt[3]{(5x+7)^2} dx = \int (5x+7)^{2/3} dx$$

$$u = 5x+7$$

$$\frac{du}{dx} = 5 \Rightarrow du = 5 dx$$

$$\int u^{2/3} du$$

$$I = \int (5x+7)^{2/3} \frac{5}{5} dx$$

$$= \frac{1}{5} \int \underbrace{(5x+7)^{2/3}}_u \underbrace{5 dx}_{du} = \frac{1}{5} \int u^{2/3} du,$$

$$= \frac{1}{5} \frac{u^{2/3+1}}{2/3+1} + C = \frac{1}{5} \cdot \frac{u^{5/3}}{5/3} + C$$

$$= \frac{3}{25} (5x+7)^{2/3} + C \quad \text{done!}$$

$$\text{EX: } I = \int 7x^2 e^{2x^3+3} dx$$

$$\int u^2 du$$

$$\int e^u du$$

$$u = 2x^3 + 3$$

$$\frac{du}{dx} = 6x^2 \Rightarrow du = 6x^2 dx$$

$$I = \frac{7}{6} \int e^{2x^3+3} \cdot \underbrace{6x^2 dx}_{du} = \frac{7}{6} \int e^u du$$

$$= \frac{7}{6} e^u + C = \frac{7}{6} e^{2x^3+3} + C \quad \text{done!}$$

$$\text{Say we had } I = \int 7x^2 e^{2x^3+3} dx$$

$$\text{Ex: } I = \int \frac{(\ln x)^5}{x} dx = \int (\ln x)^5 \underbrace{\frac{1}{x} dx}$$

$$u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x} dx$$

$$\int u^5 du$$

$$I = \int u^5 du = \frac{u^6}{6} + C = \frac{1}{6} [\ln x]^6 + C$$

$$I = \frac{1}{6} \ln x^6 + C = \frac{[\ln x]^6}{6} + C$$

$$\text{EX: } I = \int \frac{x}{x+1} dx$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \frac{a}{b+c} \text{ can't split it}$$

$$u = x+1 \Rightarrow x = u-1$$

$$\frac{du}{dx} = 1 \Rightarrow du = dx$$

$$I = \int \frac{u-1}{u} du = \int \left(\frac{u}{u} - \frac{1}{u} \right) du$$

$$= \int \left(1 - \frac{1}{u} \right) du = \int du - \int \frac{1}{u} du$$

$$= u - \ln|u| + C = x+1 - \ln|x+1| + C$$

Done!