

TRIGONOMETRIC SUBSTITUTIONS

$$\sqrt{a^2 - x^2}, \quad \sqrt{x^2 + a^2}, \quad \sqrt{x^2 - a^2}$$

$$\text{EX: } I = \int \sqrt{25 - x^2} dx \quad \boxed{x = a \tan \theta} \quad \boxed{x = a \sec \theta}$$

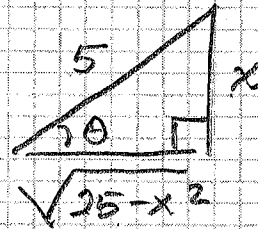
$$\boxed{x = a \sin \theta} \quad -\pi/2 \leq \theta \leq \pi/2$$

$$a = 5 \quad x = 5 \sin \theta \Rightarrow dx = 5 \cos \theta d\theta$$

$$\begin{aligned} \sqrt{25 - x^2} &= \sqrt{5^2 - 5^2 \sin^2 \theta} = 5 \sqrt{1 - \sin^2 \theta} \\ &= 5 \sqrt{\cos^2 \theta} = 5 \cos \theta \end{aligned}$$

$$\begin{aligned} I &= \int 5 \cos \theta \cdot 5 \cos \theta d\theta = 25 \int \cos^2 \theta d\theta \\ &= 25 \left[\frac{1}{2} \theta + \frac{1}{4} \sin(2\theta) \right] + C \end{aligned}$$

$$\begin{aligned} x = 5 \sin \theta &\Rightarrow \sin \theta = \frac{x}{5} \\ \theta &= \sin^{-1} \left(\frac{x}{5} \right) \end{aligned}$$



$$= 25 \left[\frac{1}{2} \sin^{-1} \left(\frac{x}{5} \right) + \frac{1}{4} \cdot 2 \cdot \frac{x}{5} \cdot \frac{\sqrt{25 - x^2}}{5} \right] + C$$

$\sin 2\theta = 2 \sin \theta \cos \theta$

$$= \frac{1}{2} \left[25 \sin^{-1} \left(\frac{x}{5} \right) + x \sqrt{25 - x^2} \right] + C$$