# MR6 Interaction Models

$$E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

$$E(Y) = -2{,}291.217 + 12.741 X_1 + 85.953 X_2$$

For $X_2 = 20$

$$E(Y) = -2{,}291{,}217 + 12.741 X_1 + 85.953 * 20$$

$$\boxed{E(Y) = -579.357 + 12.741 X_1}$$

For $X_2 = 25$

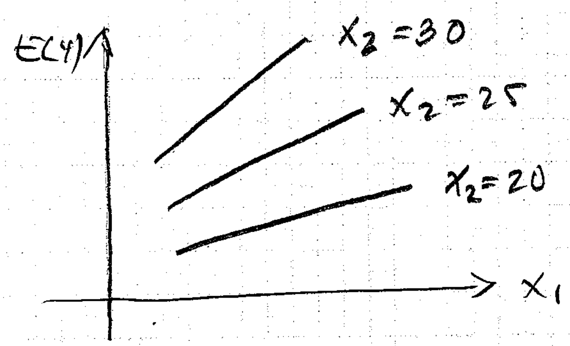
$$\boxed{E(Y) = -151.392 + 12.741 X_1}$$

For $X_2 = 30$

$$\boxed{E(Y) = 276.573 + 12.741 X_1}$$

## INTERACTION MODEL

$$E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 * X_2$$



**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .945[a] | .892 | .885 | 133.485 |
| 2 | .977[b] | .954 | .949 | 88.915 |

a. Predictors: (Constant), x2, x1

b. Predictors: (Constant), x2, x1, x1x2

**ANOVA[a]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 4283062.960 | 2 | 2141531.480 | 120.188 | .000[b] |
| | Residual | 516726.540 | 29 | 17818.157 | | |
| | Total | 4799789.500 | 31 | | | |
| 2 | Regression | 4578427.367 | 3 | 1526142.456 | 193.041 | .000[c] |
| | Residual | 221362.133 | 28 | 7905.790 | | |
| | Total | 4799789.500 | 31 | | | |

a. Dependent Variable: y

b. Predictors: (Constant), x2, x1

c. Predictors: (Constant), x2, x1, x1x2

**Coefficients[a]**

| Model | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | -2291.217 | 251.230 | | -9.120 | .000 |
| | x1 | 12.741 | .905 | .887 | 14.082 | .000 |
| | x2 | 85.953 | 8.729 | .620 | 9.847 | .000 |
| 2 | (Constant) | 489.953 | 484.808 | | 1.011 | .321 |
| | x1 | -18.590 | 5.161 | -1.294 | -3.602 | .001 |
| | x2 | -28.373 | 19.587 | -.205 | -1.449 | .159 |
| | x1x2 | 1.298 | .212 | 2.142 | 6.112 | .000 |

a. Dependent Variable: y

Test for Significant Overall Regression
(The Global Test)

$$E(y) = 489.963 - 18.590x_1 - 22.378x_2 + 1.298x_1x_2$$

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$ $\alpha = .05$

Step 1 $H_a$: At least one of the $\beta$'s is not zero

Step 2 $F = 193.041$

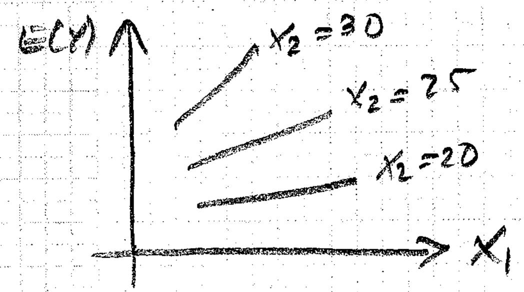
Step 3 p-value $< .001$

Step 4 Decision
Reject $H_0$

Step 5 Conclusion:
"The data provide sufficient evidence to conclude that there is a significant overall regression using all the variables in our model" or
"Our model is a useful predictor of y"

## Test for Interaction

$E(Y) = 489.953 - 18.590 X_1 - 28.373 X_2 + 1.298 X_1 X_2$




$H_0: \beta_3 = 0$

$H_a: \beta_3 \neq 0$ $\alpha = .05$

$t = 6.112$

p-value $< .001$

Reject $H_0$

"the rate of change of the mean value of Y with respect to $X_1$, changes for different values of $X_2$"

$H_a: \beta_3 > 0$

Now, Let's say we keep fixed $x_1 = 100$ and we want to estimate the change in $y$ for every one-unit increase in $x_2$

$$E(Y) = 489.953 - 18.590\,x_1 - 28.373\,x_2 + 1.298\,x_1 x_2$$

$$E(Y) = \underbrace{489.953 - 18.590 * 100}\ \underbrace{- 28.373 x_2 + 1.298 * 100 * x_2}$$

$$E(Y) = -1{,}369.047 + 101.427\,x_2$$

Estimated $x_2$ slope $= 101.427$

<u>Last comment</u>

Since interaction between $x_1$ and $x_2$ was found to be statistically significant, we should not conduct tests on $x_1$ and $x_2$ and we should keep them both in our model