INTRODUCTION TO PROBABILITY

Probability ~ Percentage

.50 50% chance

$$\boxed{0 \leq p \leq 1}$$

$p = 0$ Impossible event

$p = 1$ Sure event

$$\boxed{\Sigma p = 1}$$

$p(\text{rain}) = .4$

$p(\text{not rain}) = .6$

Experiment activity with an observable result

Trials repetitions of the experiment

Outcomes the results of each trial

Sample Space Set of all possible outcomes

Sample Points elements of the sample space

Event subset of the sample space

Example: Flipping a coin three times

Head = H Tail = T

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Event A: getting exactly two heads

$A = \{HHT, HTH, THH\}$

$n(A)$ = number of elements in A

$n(S)$ = " " " " S

If all the elements in the Sample Space S are equally likely

$$\boxed{P(A) = \frac{n(A)}{n(S)}}$$

Example: Bag of marbles

4 Red
6 Black
10

$$P(\text{Red}) = \frac{n(\text{Red})}{n(S)} = \frac{4}{10} = .4$$

Example: Deck of 52 cards, 4 suits

Red: Hearts, Diamonds — Black: Clubs, Spades

Hearts: A, 2, 3, ..., 10, J, Q, K (Face) — 13

Diamonds: A, 2, 3, ..., 10, J, Q, K (Face) — 13

Clubs — 13

Spades — 13

$13 + 13 + 13 + 13 = 52$

$$P(\text{Jack}) = \frac{n(\text{Jacks})}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{Red}) = \frac{26}{52} = \frac{1}{2} = .5$$

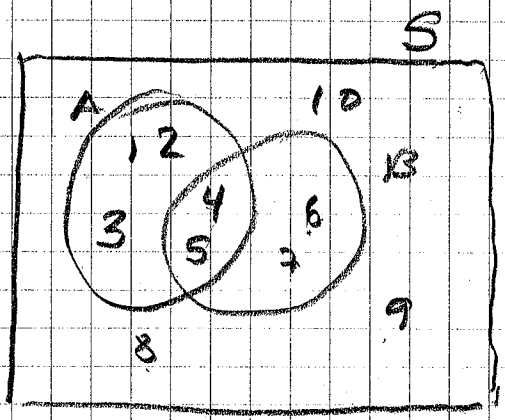
Addition Rule for Probability

$A = \{1, 2, 3, 4, 5\}$

$n(A) = 5$

$B = \{4, 5, 6, 7\}$

$n(B) = 4$



$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ $\quad n(A \cup B) = 7$

$A \cap B = \{4, 5\}$ $\quad n(A \cap B) = 2$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$7 = 5 + 4 - 2$ ✓

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

If $A \cap B = \emptyset$ then $P(A \cap B) = 0$ and

$$\boxed{P(A \cup B) = P(A) + P(B)}$$ for Disjoint sets (or Mutually exclosive events)