

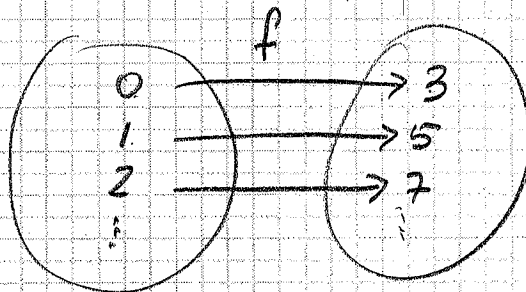
INVERSE FUNCTIONS

Example: Let f be the function $f(x) = 2x + 3$

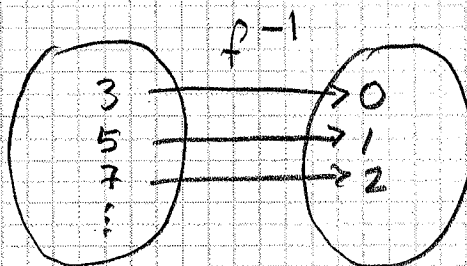
$$f(0) = 2 \cdot 0 + 3 = 3$$

$$f(1) = 2 \cdot 1 + 3 = 5$$

$$f(2) = 2 \cdot 2 + 3 = 7$$



the inverse of f
would be:



$$y = 2x + 3$$

swap
 x, y

$$x = 2y + 3$$

solve
for y

$$x - 3 = 2y$$

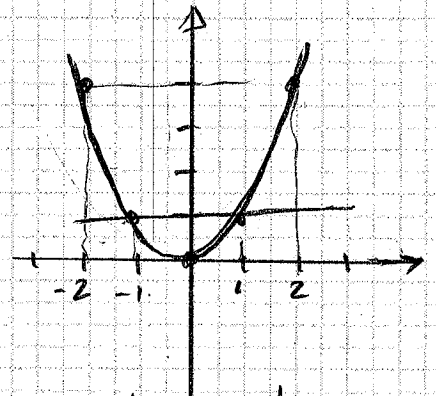
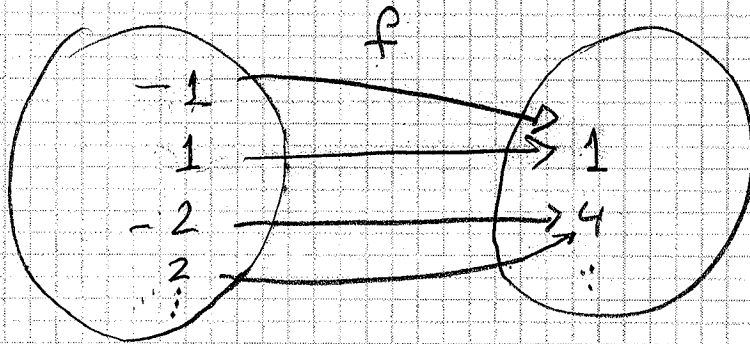
$$\frac{x-3}{2} = y$$

$$f^{-1}(x) = \frac{x-3}{2}$$

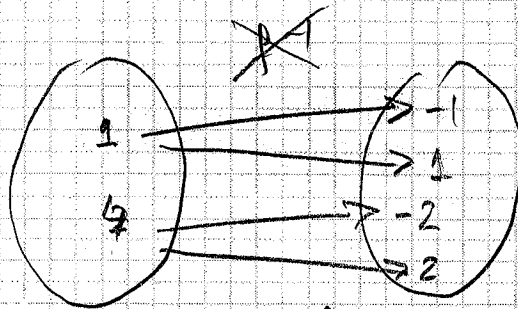
$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{x-3}{2}\right) = 2 \cdot \frac{x-3}{2} + 3$$

$$= x - 3 + 3 = x$$

Example $f(x) = x^2$



not one-to-one



not a function

Example: $f(x) = x^2$ restricted to $[0, +\infty)$

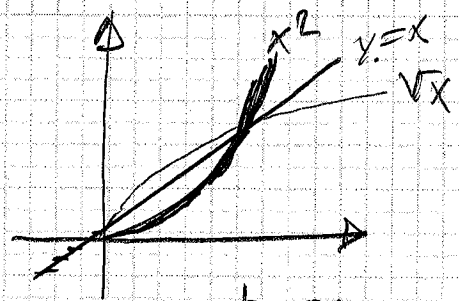
$$y = x^2 \quad x \geq 0$$

$$x = y^2 \quad y \geq 0$$

$$y^2 = x \quad y \geq 0$$

$$y = \sqrt{x}$$

swap
x and y



one-to-one

$$y^2 = a^2 \Rightarrow \begin{cases} y = a \\ \text{or} \\ y = -a \end{cases}$$

Example: $f(x) = \frac{3x-1}{x+1}$ $x \neq -1$
Find f^{-1}

$$y = \frac{3x-1}{x+1} \quad x \neq -1$$

swap x, y $x = \frac{3y-1}{y+1}$ $y \neq -1$

solve for y $x(y+1) = 3y-1$

$$xy + x = 3y - 1$$

$$x+1 = 3y - xy$$

$$x+1 = y(3-x)$$

$$\frac{x+1}{3-x} = y$$

$$f^{-1}(x) = \frac{x+1}{3-x} \quad \text{for } x \neq 3$$

CHECK

$$\begin{aligned} (f \circ f^{-1})(x) &= f(f^{-1}(x)) = f\left(\frac{x+1}{3-x}\right) = \frac{3 \cdot \frac{x+1}{3-x} - 1}{\frac{x+1}{3-x} + 1} = \\ &= \frac{\frac{3x+3 - (3-x)}{3-x}}{\frac{(x+1) + (3-x)}{3-x}} = \frac{\frac{3x+3-3+x}{3-x}}{\frac{x+1+3-x}{3-x}} = \frac{\frac{4x}{3-x}}{\frac{4}{3-x}} = \frac{4x}{3-x} \cdot \frac{3-x}{4} = x \end{aligned}$$

$$(f^{-1} \circ f)(x) = \dots = x$$