JOINT CUMMULATIVE DISTRIBUTION FUNCTIONS

Discrete

joint probability mass function
pmf

$$f(x_i, y_j) = P(X = x_i, Y = y_j)$$

joint cummulative distribution function (joint cdf)

$$F(x,y) = P(X \le x, Y \le y)$$

$$F(x,y) = \sum_{x_i \le x} \sum_{y_j \le y} P(x_i, y_j)$$

Example

x \ y	0	1
1	1/12	1/12
2	1/12	1/12
3	1/12	1/12
4	1/12	1/12
5	1/12	1/12
6	1/12	1/12

$$F(3,1) = \sum_{x_i \le 3} \sum_{y_j \le 1} P(x_i, y_j)$$

$$= 6 * \frac{1}{12} = 1/2$$

Continuous

joint probability density function
pdf

$$f(x,y) = P[(x,y) \in A] =$$

$$= \iint_A f(x,y)\, dx\, dy$$

joint cummulative distribution function (joint cdf)

$$F(x,y) = P(X \le x, Y \le y)$$

$$F(x,y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(s,t)\, ds\, dt$$

if $-\infty \le x \le \infty$ and $-\infty \le y \le \infty$

if $a \le x \le b$ and $c \le y \le d$

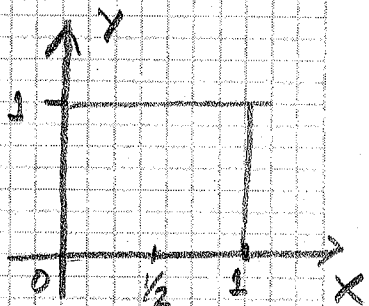
$$F(x,y) = \int_{c}^{y} \int_{a}^{x} f(s,t)\, ds\, dt$$

Example:

x, y in $[0,1]$

$f(x,y) = 4xy$

$$\int_0^1 \int_0^1 f(x,y)\, dx\, dy = 1$$



$$F(1/2, 2/3) = P(X \le 1/2, Y \le 2/3)$$

$$\int_0^{2/3} \int_0^{1/2} f(x,y)\, dx\, dy = \int_0^{2/3} \int_0^{1/2} 4xy\, dx\, dy$$

$$= \int_0^{2/3} 4y \int_0^{1/2} x \, dx \, dy = \int_0^{2/3} 4y \left[\frac{x^2}{2} \right]_0^{1/2} dy =$$

$$= \int_0^{2/3} 2y \left[x^2 \right]_0^{1/2} dy = \int_0^{2/3} 2y \left[\frac{1}{4} - 0 \right] dy = \int_0^{2/3} \frac{y}{2} \, dy$$

$$= \frac{1}{2} \left[\frac{y^2}{2} \right]_0^{2/3} = \frac{1}{4} \left[\left(\frac{2}{3} \right)^2 - 0 \right] = \frac{1}{4} \cdot \frac{4}{9} = \frac{1}{9}$$