JOINT PROBABILITIES

Definition: Let X be a discrete random variable with values $\{x_1, x_2, \ldots, x_n\}$ and Y a discrete random variable with values $\{y_1, y_2, \ldots, y_m\}$. The ordered pair (X, Y) of random variables, therefore, has values $\{(x_1, y_1), (x_1, y_2), \ldots, (x_n, y_m)\}$. The joint probability distribution (or joint probability mass function) of X and Y is the function

$$f(x_i, y_j) = P(X = x_i, Y = y_j)$$

Example Let's say we roll a dice first and then flip a coin, and let X be the random variable for the outcome of the die and Y for the coin.

$$\text{pmf}(3, T) =$$
$$= P(X=3, Y=T)$$
$$= P(X=3) \cdot P(Y=T)$$
$$= \frac{1}{6} * \frac{1}{2} = \frac{1}{12}$$

X \ Y	H	T
1	1/12	1/12
2	1/12	1/12
3	1/12	1/12
4	1/12	1/12
5	1/12	1/12
6	1/12	1/12

Joint Probability Table

$$0 \leq P(x_i, y_j) \leq 1$$

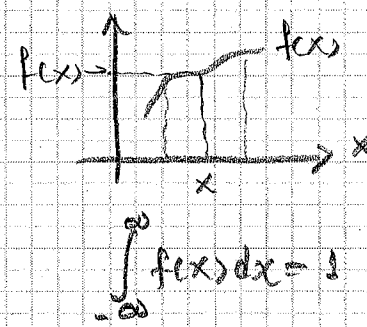
$$\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) = 1$$

Y	0	1
P	1/2	1/2

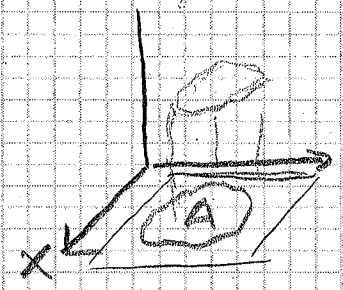
Definition: Let X and Y be continuous random variables with values in $[a,b]$ and $[c,d]$ respectively. The joint probability density function (pdf) of X and Y is a function

$$f(x,y) = P[(X,Y) \in A] = \iint_A f(x,y)\,dx\,dy$$

for any region A in the x-y plane



$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$



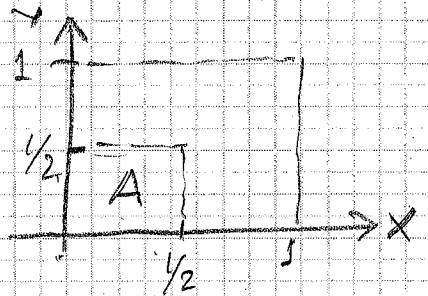
$$\int_c^d \int_a^b f(x,y)\,dx\,dy = 1$$

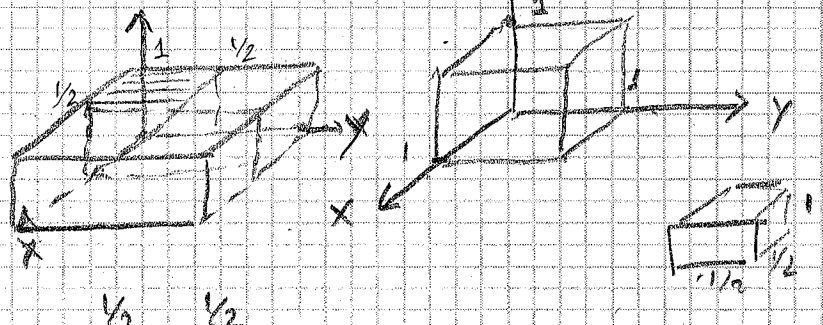
volumes

Example: X and Y have values in $[0,1]$ and their joint pdf is $f(x,y) = 1$

$$\int_0^1 \int_0^1 f(x,y)\,dx\,dy = \int_0^1 \int_0^1 dx\,dy = \int_0^1 x\Big|_0^1 dy =$$

$$= \int_0^1 dy = y\Big|_0^1 = 1$$





$$P[(X,Y) \in A] = \int_0^{1/2} \int_0^{1/2} 1 \cdot dx\,dy = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$